

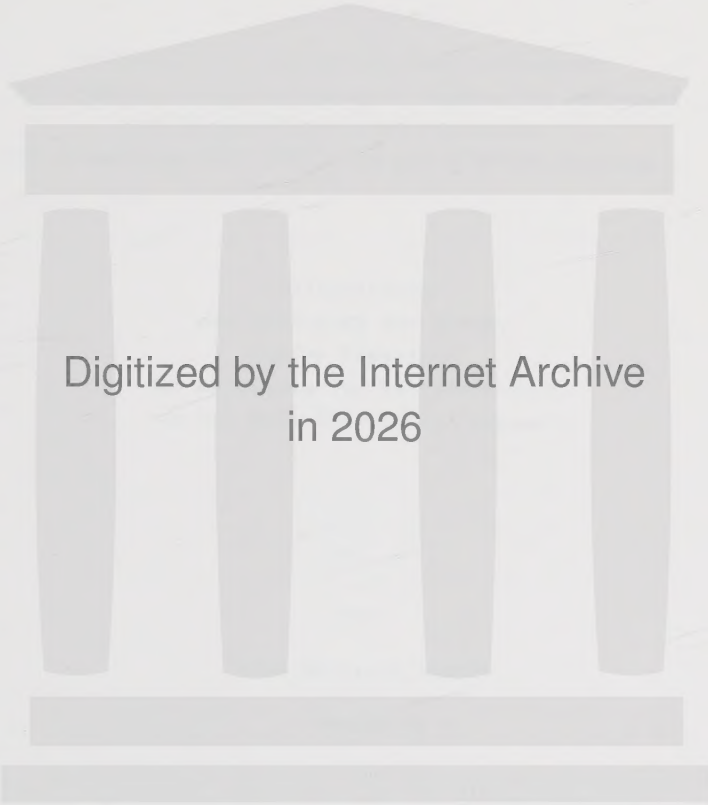
Optimal low-sensitivity filters for the recovery of sampled signals in baseband or frequency-shifted position, with applications to PCM/audio and PCM/FDM conversion

Optimale Filter niedriger Empfindlichkeit zur Rückgewinnung abgetasteter Signale in Basisband- und frequenzversetzter Position, mit Anwendungen auf PCM/Audio und PCM/FDM Umsetzung

Dissertation
zur Erlangung eines Grades
Doktor-Ingenieur
der Abteilung für Elektrotechnik
an der Ruhr-Universität Bochum

von
Anis Guirguis Khalil
Alexandria

Bochum
1979



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Summary

With the increasing use of digital technology for signal processing, the importance of conversion circuits, which generate a continuous signal from a modulated impulse sequence and vice versa, is increasing. Such circuits can be used e.g. in PCM terminal equipment, PCM/FDM transmultiplexers, or if a signal is processed by a digital filter or a switched-capacitor filter.

In this thesis adequate circuits are analyzed with the help of results from the resonant-transfer theory. In this way the analysis, optimization and interpretation of results are advantageously simplified. The design of these networks is the result computer programs that can calculate the optimum network parameters. The resulting networks enable almost complete conversion of the available energy into the desired frequency band. A $(\sin x/x)$ correction is not required. The circuits exhibit the desired minimal sensitivity with respect to variations in element values, which allows meaningful utilization in practice.

Two design examples are given, namely, in the case of PCM recovery and in the case of PCM/FDM transmultiplexer. Complete structures with element values, sensitivity analysis and practical measurement results are presented. The results achieved for PCM recovery are superior to all previously published results, while results for the PCM/FDM transmultiplexer are the first to be published.

Zusammenfassung

Mit zunehmendem Einsatz der Digitaltechnik in der Signalverarbeitung nimmt die Bedeutung von Umwandlungsschaltungen zu, die aus einer modulierten Impulsfolge ein zeitkontinuierliches Signal erzeugen und umgekehrt. Solche Schaltungen können z. B. erforderlich sein bei PCM-Endstellen, PCM/FDM Transmultiplexern oder, wenn ein Signal durch Digitalfilter oder Schalter-Kondensator-Filter verarbeitet wird.

In der vorliegenden Arbeit werden entsprechende Schaltungen mit Hilfe von Ergebnissen aus der Theorie des Resonanztransfers analysiert. Auf diese Weise werden die Durchführung der Analyse und Optimierung sowie die Bewertung der Ergebnisse vorteilhaft einfach gestaltet. Dem Entwurf dieser Schaltungen liegt ein Programm zugrunde, das optimale Schaltungen berechnen kann. Die so erhaltenen Schaltungen ermöglichen es, die verfügbare Leistung praktisch vollständig in den jeweils gewünschten Frequenzbereich zu übertragen. Eine $(\sin x/x)$ -Korrektur ist nicht erforderlich. Die Schaltungen weisen die erwünschte minimale Empfindlichkeit gegen Abweichung der Bauelementwerte auf, was den sinnvollen Einsatz in der Praxis erst ermöglicht.

Es werden zwei Entwurfsbeispiele angegeben, und zwar für den Fall einer PCM-Rückgewinnungsschaltung und für den Fall eines PCM/FDM Transmultiplexers. Komplette Strukturen mit Bauelementwerten, Empfindlichkeitsuntersuchungen und vor allem praktische Meßergebnisse werden angegeben. Die erzielten Ergebnisse für PCM-Rückgewinnungsschaltung übertreffen alle anderen bisher veröffentlichten Ergebnisse, während Ergebnisse für den PCM/FDM Transmultiplexer hier überhaupt zum ersten Mal erscheinen.

Chapter 1

Introduction

This thesis presents a unique solution to various important problems of modern telephone systems, where signals available in discrete time form (as pulse amplitude modulation) have to be converted to continuous time signals for further processing or for transmission purposes. A classic case is encountered at the PCM receiving end where a decoded PCM bit stream has to be converted to voice frequency (basic-band).

Another problem dealt with in this work is that of transmultiplexing a PCM system with an FDM one. Direct conversion of decoded PCM signals into an FDM channel is shown to be possible and the direct sampling of an FDM signal to generate PCM signals presents a solution for the direct FDM/PCM conversion.

Several similar problems are also discussed in this work such as anti-aliasing filters needed e.g. in an all digital transmultiplexer and digital SSB generator for FDM systems. Anti-aliasing circuits needed for switched capacitor filters can be also designed in the same way presented in this thesis.

Analysis of circuits for conversion of discrete time signals into continuous time ones corresponds to an analysis of switched filters, i.e. of those filters containing periodically operated switches. It is the pulse impedance theory which made it possible to calculate the optimum solutions for the practical problems mentioned above, and many other problems. This theory enables exploiting well understood theories and principles known in the fields of circuit theory and computer aided design (CAD). In view of its importance in our work, the pulse impedance theory is presented in detail in Chapter 2. A complete analysis of the conversion circuits and some aspects of the ideal universal filter are also given in Chapter 2.

In Chapter 3, the solution strategy for the problem of designing optimal low-sensitivity conversion circuits is presented. Theoretical as well as practical results are given in Chapter 4 together with a sensitivity analysis which confirms the optimal

behavior of the results obtained. At the end of every chapter a separate reference list is given.

The following sections of Chapter 1 are reserved for discussing the FDM and PCM types of multiplexing and for reviewing the solutions available for PCM recovery and PCM/FDM transmultiplexing. Other applications of the conversion circuits are also mentioned.

1.1 Frequency Division Multiplex | 1,2,3,4 |

As the most widely used method worldwide, the frequency division multiplex (FDM) method of transmitting many telephone calls through the same transmission medium, represents the main member of continuous time modulation family. Frequency translation of the various voice channels is achieved by balanced modulators (e.g. ring modulators) together with channel filters to choose one of the side-bands appearing at the output of the modulator. The signal gained is then of the single sideband suppressed carrier (SSBSC) type.

Since the effective bandwidth of the transmitted voice signals is 3.1 kHz, a standard spacing of SSB channels has been set at 4 kHz. This would allow for a reasonable separation between the adjacent channels using economically designed channel filters. The number of channels that can be transmitted between two points depends mainly upon the transmission medium, but for purpose of compatibility, a universal multiplexing system is used. A basic "primary group" containing 12 channels and occupying in its basic location the frequency range between 60 kHz and 108 kHz is an international standard. A basic secondary group is composed of 5 primary groups (60 channels) and occupies the frequency range 312 - 552 kHz. The complexity of the system can grow to handle at present up to 10800 channels.

As the requirements for the channel filters are stringent, the filter design problem suggests the modulation technique used to build a basic primary group. The modulation is often done in two steps. In the "pregroup" modulation approach, 3 voice signals are modulated by carriers of 12,16, and 20 kHz and the upper side-bands are chosen by the use of LC channel filters as shown in Fig. 1. This way the design of the required LC channel filters is

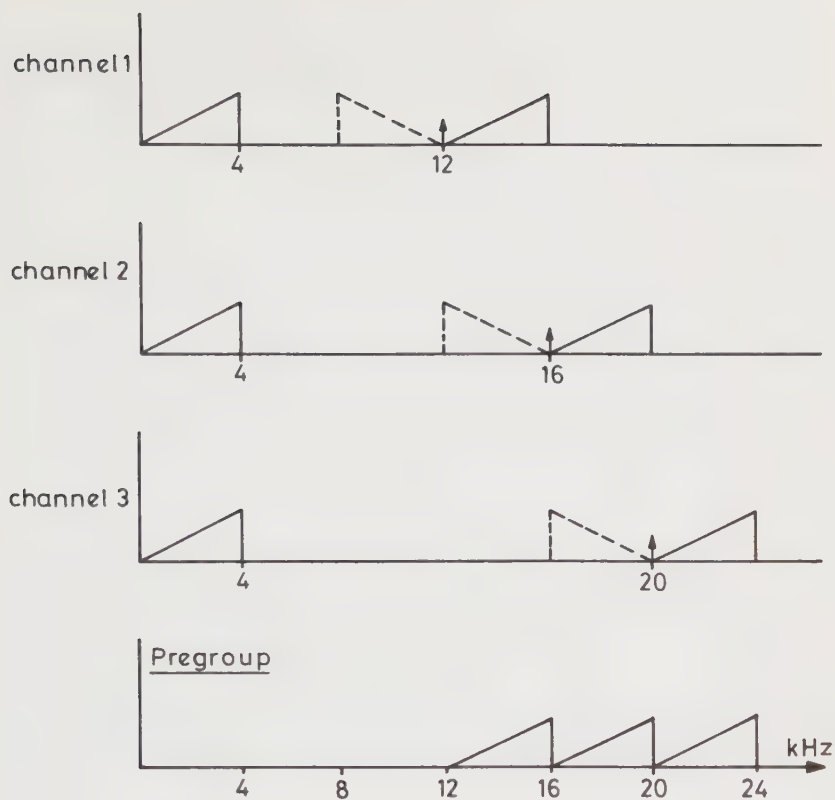


Fig. 1.1 Modulation scheme for a pregroup

reduced to a reasonable complexity. Four pregroups are then transferred to the 60 - 108 kHz basic primary group location by carriers of 84, 96, 108, and 120 kHz and choosing the lower sidebands. The filter problem in the second modulation step is largely less stringent than in the first step.

With the recent development in the field of monolithic quartz filters and mechanical filters new modulation approaches was suggested. Every voice channel can be modulated e.g. first by 48 kHz carrier and 12 mechanical filters are used to choose the upper sidebands located in the frequency range 48.3 to 51.4 kHz. In a second modulation step, the 12 channels are shifted to the individual channel positions of the basic primary group (60 to 108 kHz) by the aid of 12 carrier signals 112, 116, ..., 156 kHz.

In the further multiplexing steps, 5 basic primary groups form a basic supergroup (60 channels occupying the frequency range 312 - 552 kHz); 5 basic supergroups form a basic mastergroup (300 channels), 3 basic mastergroups form a basic supermastergroup (900 channels). A 60 MHz system containing 10800 channels (12 basic super mastergroups) is now available [4].

1.2 Pulse-Code Modulation [1,3,4,5,6,7]

Pulse-code-Modulation (PCM), invented by Allan H. Reeves in 1937, is now the primary method seriously considered for time division multiplexing. This stems from the resistance of PCM to interference and noise. We are going to consider the PCM as a secure means of transmitting samples of several signal channels multiplexed in time. In other words, the effects of quantizing as well as encoding and decoding processes will not be considered in our work.

Specifically, we will examine the conversion of the signal-samples (which are of discrete time nature) into a continuous time signal. Samples taken from a signal occupy only a small fraction of time, allowing insertion of many other samples from other signals between repetitive sampling periods.

The sampling theorem - which is the central idea of the time division multiplex - states in its simplest form that "a band-limited signal which has no spectral components above frequency f_c Hz is uniquely determined by its values (samples) at uniform intervals less than $\frac{1}{2f_c}$ seconds apart". A bandlimited signal can be expressed as a time function $f(t)$ whose Fourier transform $F(j\omega)$, given by

$$F(j\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt, \quad (1.1)$$

is zero for all values of ω greater than the cutoff frequency ω_c i.e.

$$F(j\omega) = 0, \quad |\omega| > \omega_c \quad (1.2)$$

with

$$\omega_c = 2\pi f_c. \quad (1.3)$$

Sampling the signal $f(t)$ uniformly at T second intervals amounts, ideally, to multiplying $f(t)$ by a uniform impulse train, the sampled function $f_s(t)$ is now given by

$$f_s(t) = \sum_{n=-\infty}^{\infty} f(t) \delta(t-nT) \quad (1.4)$$

with the choice of the sampling period T such that the sampling frequency Ω satisfies

$$\Omega = \frac{2\pi}{T} \geq 2\omega_c, \quad (1.5)$$

and the resulting Fourier transform of the sampled signal is now

$$F_s(j\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(j\omega - jn\Omega). \quad (1.6)$$

It is "theoretically" guaranteed to recover the original signal $F(j\omega)$ and $f(t)$ by passing the sampled signal through an ideal low-pass filter whose cutoff frequency is the same as that of the original signal (f_c).

1.2.1 Practical aspects

The designer faces three main imperfections when dealing with realizations of practical time division multiplex systems. First, a true bandlimited signal does not exist because it neces-

sitates simply a signal of infinitely long duration. Secondly, practical sampling produces samples of finite duration and shape rather than ideal impulses. The third imperfection is that of the ideal low pass filter required for signal recovery. In practical systems (PCM) the incoming voice signals are "band-limited" by suppressing sufficiently all frequencies higher than 3.4 kHz and then sampling the output filtered signals at 8 kHz sampling rate. An attenuation of about 30 dB is considered a good compromise between the amount of overlapping (aliasing) noise and the complexity of used filters [6,7] for both the transmit and receive (recovery) filters.

1.2.2 Pulse-Amplitude-Modulation

It was pointed out that in practical time division multiplexing systems, the samples taken from signals are not ideal impulses. The finite width of the samples results indifferent spectral density of the sampled signal than that of equation (1.6). To study this effect, assume that the sampling is performed by periodically rectangular pulses of width τ seconds and height A , repeating every T seconds (it will be always assumed that T satisfies equation (1.5) unless otherwise stated). We shall denote this pulse train by $p_\tau(t)$. $P_\tau(j\omega)$, the spectrum of the pulse train is

$$P_\tau(j\omega) = \mathcal{F}\{p_\tau(t)\} = A\Omega \sum_{n=-\infty}^{\infty} \text{Sa}\left(\frac{n\tau\Omega}{2}\right) \delta(\omega - n\Omega) \quad (1.7)$$

with

$$\text{Sa}(x) = \frac{\sin(x)}{x} \quad (1.8)$$

Now, the sampled function

$$y_s = f(t)p_\tau(t) \quad (1.9)$$

is composed of a pulse train whose top is shaped with the corresponding value of $f(t)$. Hence $Y_s(\omega)$, the spectrum of $y_s(t)$, is obtained by convolving $F(\omega)$ with $P_\tau(\omega)$.

$$Y_s(j\omega) = \frac{1}{2\pi} F(j\omega) * P_\tau(j\omega) \quad (1.10)$$

$$= \frac{A}{T} \sum_{n=-\infty}^{\infty} \text{Sa}(n\tau\Omega/2) F(j\omega - n\Omega) \quad (1.11)$$

The spectrum (1.11) represents the spectrum $F(\omega)$ repeating itself every Ω radians per second, but with amplitude varying as $S_a(n\tau\Omega/2)$. The samples need not to be of rectangular shape but they may assume any pulse form $q(t)$. This will merely cause the shape of the spectrum to change according to the corresponding $Q(j\omega)$ where

$$Q(j\omega) = \mathcal{F}\{p(t)\} \quad (1.12)$$

The original signal can be recovered from the sampled signal $f_s(t)$ exactly in the same way as in the case of ideal sampling by impulses, namely by using a low-pass filter with cutoff at ω_c . This way of sampling is called natural sampling since the sampled signal essentially represents the periodic pulse train whose amplitude is modulated by $f(t)$.

Another way of sampling is called instantaneous sampling, where all the pulses in the sampled signal have the same form but their amplitudes are proportional to the corresponding signal value. This way of sampling, called instantaneous sampling, because the individual samples carry only information about the instantaneous value of $f(t)$, in contrast to natural sampling where the individual samples carry information about $f(t)$ over the entire width of each sampling pulse. The pulses may assume any general form $q(t)$ and the sampled signal will be denoted by $x_s(t)$. Thus, for instantaneous sampling

$$x_s(t) = \sum_{n=-\infty}^{\infty} f(nT) q(t-nT) \quad (1.13)$$

This signal can be considered as the output of a system whose impulse response

$$h(t) = q(t) \quad (1.14)$$

when this system is fed by the impulse train f_s of equation (1.4). Thus if $q(t)$ has the spectrum $Q(j\omega)$ then,

$$X_s(j\omega) = \mathcal{F}\{x_s(t)\} = F_s(j\omega) Q(j\omega) \quad (1.15)$$

where $F_s(\omega)$ is the Fourier transform of $f_s(t)$. Using equation (1.6), we get

$$X_S(j\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} Q(j\omega) F(j\omega - jn\Omega) . \quad (1.16)$$

The spectrum of the sampled function $x_s(t)$ consists of periodic repetition of $F(j\omega)$ which is multiplied by $Q(j\omega)$. At each frequency there is a different multiplication factor. This is an essential difference to the case of natural sampling. This makes the recovery process different from that used in case of natural sampling. If we use a low-pass filter with a cutoff frequency ω_c , the output spectrum will be

$$F(j\omega) Q(j\omega) . \quad (1.17)$$

The desired signal $f(t)$ can be recovered from this output by transmitting it through another filter whose transfer function is $1/Q(j\omega)$ only over the range $(0, \omega_c)$. Outside this range it may be chosen to be arbitrary. If both filters, namely the low pass and correction $1/Q(j\omega)$ are combined in a composite filter, the composite transfer function is then

$$H(\omega) = \begin{matrix} 1/Q(j\omega) & |\omega| < \omega_c \\ 0 & 0 \text{ otherwise} \end{matrix} . \quad (1.18)$$

A famous example of such recovery filters used in practice, is the low-pass filter and the correction $X/\sin(X)$ used in conjunction with a sample and hold circuit for PCM recovery [9,10]. In such a case, the samples are T seconds wide and the value of X in the correction term is equal to $\pi \frac{\omega}{\Omega}$. Besides necessitating more circuit elements to design the composite filter whose transfer function now corresponds to equation (1.18), it is to be noted that inside the pass band there is a difference of 2.75 dB in the filter gain.

Alternatively, $f(t)$ may be recovered from $x_s(t)$ by multiplying $x_s(t)$ by $\delta_T(t)$, a train of impulses. This obviously yields $f_s(t)$. The desired signal $f(t)$ can now be recovered from $f_s(t)$ by using an ideal low pass filter without the correction $1/Q(j\omega)$. In practice, the stair-case signal of the sample and hold circuit can be resampled by narrow pulses and fed to a filtering circuit which

does not need any correction as long as the samples are of sufficiently small width in the order of 1 or 2 % of the period T .

1.2.3 Resonant-transfer principle

The trade off's between wide or narrow pulses for implementing a practical system includes also the problem of the energy contained in the pulses to be filtered to recover the signal $f(t)$ or to choose a higher side band [11,12] using a band pass filter. It is always desirable that the signal power level be kept sufficiently high to avoid the noise effects of passive and active elements used in implementing the recovery circuits. Another factor affecting the choice of pulse width is the constraints put on the pulse width of individual signal samples in a time multiplex system. An efficient solution to the problem of energy transfer using narrow pulses was independently invented by Haard and Svala, Cattermore, Lewis and French [6], by taking a sample dependent not only on the instantaneous amplitude of the signal but also on signal energy accumulated during the interval between samples. As an example we consider the PAM channel shown in Fig.2 containing two identical filters and a pulse path composed of the switch and an inductance $2L$. The capacitances C_1 and C_2 of the two filters are equal in value,

$$C_1 = C_2 = C. \quad (1.19)$$

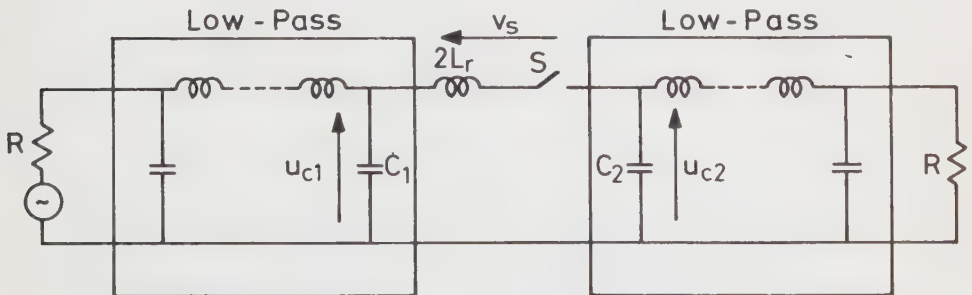


Fig. 1.2 PAM resonant-transfer arrangement

The storage element C_1 on the left hand side is charged slowly by a substantially smooth flow of power from the signal source. On closure of the switch S , C_1 is discharged rapidly into the pulse path ($2L_r, S$), all the stored energy being trans-

ferred to C_2 in a relatively short period if the sampling time is equal to half the resonant period of the circuit of Fig. 1.3 namely

$$\tau = \pi \sqrt{L_r C} \quad (1.20)$$

if τ is sufficiently smaller than the sampling period T and all other time constants and resonant periods of the low pass filters.

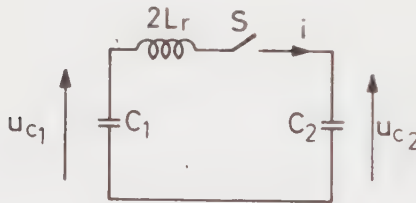


Fig. 1.3 Resonant-transfer arrangement

The current i flowing through the resonant transfer arrangement of Fig. 1.3 is given by

$$i = \frac{v_1 - v_2}{2\omega_0 L_r} \sin \omega_0 t \quad (1.21)$$

with

$$\omega_0 = \frac{1}{\sqrt{L_r C}} \quad (1.22)$$

and v_1, v_2 the voltages across C_1, C_2 before switch closure. The voltages across C_1, C_2 during switch closure are given by

$$u_{C_1} = \frac{1}{2}(v_1 + v_2) + \frac{1}{2}(v_1 - v_2) \cos \omega_0 t \quad (1.23)$$

$$u_{C_2} = \frac{1}{2}(v_1 + v_2) + \frac{1}{2}(v_2 - v_1) \cos \omega_0 t \quad (1.24)$$

In all the foregoing equations, the time t is measured from the moment at which the switch is closed. When S opens again after τ seconds, as given by equ. (1.20), we get

$$i = 0 \quad (1.25)$$

$$u_{C_1} = v_2 \quad (1.26)$$

$$u_{C_2} = v_1 \quad (1.27)$$

and the energy exchange takes place. C_2 now discharges slowly to yield a substantially smooth flow of power into the load. A lossless transmission is possible by perfect design of true low pass filters on both sides of the sampling arrangement.

The analysis and applications of the resonant transfer principle and the related problems of designing ideal and practical filtering circuits was the subject of many publications among which the works of Cattermole [6,13] and Kraus [14]. Complete analysis [15,16] and many applications possibilities were worked out by Fettweis [11,12,17-22]. Based on these works, the optimum solutions for symmetrical resonant transfer circuits were worked out [23].

1.2.4 Multiplexing of PCM systems

Samples of different signals (channels) obtained in the manner discussed above are quantized and coded through an A/D converter which delivers an 8 bit/word companded PCM. The multiplexing process can take place either before or after the coding process [24]. The channels are grouped in 30 channel groups in Europe or 24 channel groups in the USA and Japan to build a primary group. According to the different choices in the American and European systems, different system hierarchies were recommended by the CCITT. In the European system, 4 primary groups form a 120 channel secondary group with a 8.448 Mbit/s transmission rate while in the American system 6.312 Mbit/s rate is required to build a 96 channel secondary group from 4 primary groups.

1.3 Recovery of PCM signals

On the receiving end of a PCM channel (or channel bank), the incoming PCM signals are decoded and converted to PAM signals through a digital to analog converter. The PAM signal is passed through an interpolating system to recover the original continuous time signal. Theoretical aspects of this process were discussed in section 1.2 and the practical implementation of various systems will be discussed in this section.

1.3.1 Use of RC active filters

System specifications and detailed design of an interpolating system are described in [8] and [9]. The system itself can be seen

to be composed of 2 parts:

1. Sample and hold circuit,
2. low-pass filter .

In [8], the sample and hold circuit is build as a part of the 5th order low-pass filter to reduce the number of operational amplifiers used to 2. The whole circuit is optimized to meet the overall requirement of pass band ripple and stop band lowest attenuation. In [9], however, another construction with 8 operational amplifiers was optimized to attain reasonable sensitivity performance. The optimized solution produces an overall low pass characteristic of the sample and hold-filter combination.

1.3.2 Use of LC filters

The requirement of at least 25 dB suppression of the unwanted side bands, suggested the use of LC filters with 2 coils [7,24-26], if the number of channels to be used is not too large. The recovery system itself is composed of 3 parts:

1. Sample and hold circuit,
2. low pass filter,
3. correction circuit.

An example of such a system is given by Kliem [26,27] and is shown in Fig. 1.4.

The first amplifier is a buffer stage between the SH (Sample & Hold) circuit and the LC filter. The filter itself is a doubly terminated low pass Cauer filter of the 5th order with 2 zeros of transmission. The output of the filter is fed to the correction circuit which compensates for the $\sin x/x$ effect of the SH circuit. An example for the correction circuit is shown in Fig. 1.5 [5].

We present in Chapter 4 a complete scheme for PCM recovery for 30 channels using only one driving stage shared by the 30 channels, thus minimizing the active elements and their dissipation. The scheme is also superior to both systems discussed in this section in many other aspects.

1.4 FDM/PCM Transmultiplexers

With the increased use of the two types of multiplexing, namely, frequency division multiplexing (FDM) and time division multiplexing (PCM), the need arose for an interconnecting system between both systems. The problem is evident to be one of the several discrete-time/continuous-time conversion problems encountered in communication engineering.

The method most simple to understand is that of reducing both types of multiplexed signals (channels) to voice signals and then converting them to the other multiplexing format. This method, usually called back-to-back-method, is the one presently used in practice.

The search for alternative solutions was, and still is, the subject of several researches. This alternative solution has to compete with the back to back method economically. On the other hand, the direct conversion is evident from the properties of FDM signals and PAM signals (obtained from decoding PCM signals). The PAM signals have theoretically infinite number of side bands and it is possible to choose a single side band by the proper design of a band pass filter. The sampling of an FDM channel at an 8 kHz rate is equivalent to sampling the original voice signal because of the limited band nature of a FDM channel. Detailed solution to the design problem of the band-pass circuits used for this purpose is presented in this thesis.

Another approach, exclusively based on the use of digital signal processing, was activated in the last ten years by the revolution in manufacturing electronic circuits used in implementing digital filters and other processing devices by large scale integration (LSI). A digital transmultiplexer receives at the PCM side two 30-channel PCM groups and processes them to produce a complex digital signal, which, when decoded and filtered by a suitable band pass filter, produces one basic secondary group in the 312 - 552 band. In the other direction, a basic secondary group is sampled at e.g. 576 kHz rate and passed to an A/D converter and presented to the digital processor to produce at the output end 60 PCM channels. For such a transmultiplexer the discrete-time/continuous-time conversion is unavoidable. We propose the use of the design method of this thesis for such conversions.

The detailed description of the suggested designs of the digital transmultiplexer goes beyond the scope of our work and will not be treated here [28-38].

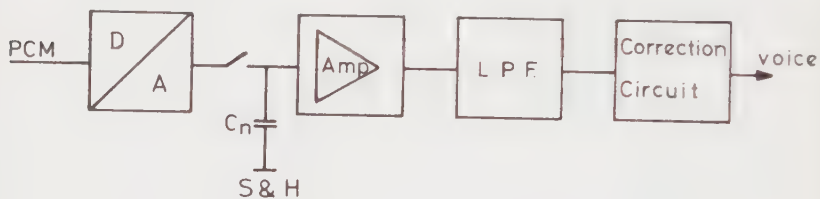


Fig. 1.4 PCM-recovery arrangement

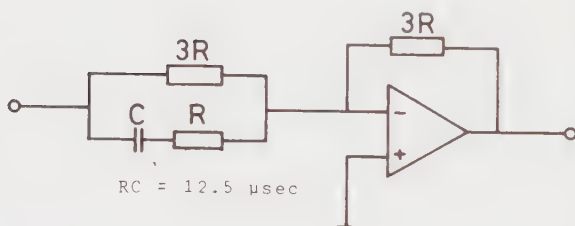


Fig. 1.5 $(\sin X/X)$ correction circuit

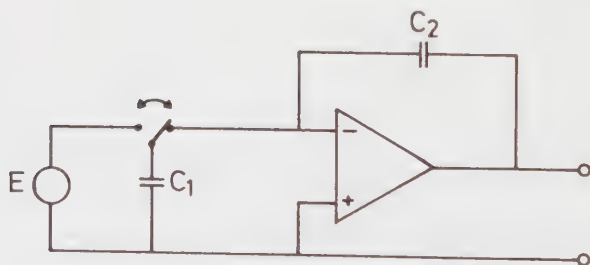


Fig. 1.6 Switched-capacitor integrator

1.5 Switched capacitor filters

The subject of switched-capacitor filters is closely related to the switched circuits and switched filters for two reasons: First, the switched capacitor filters are themselves switched filters and can be analyzed and designed with established known methods used in treating switched filters [18,21,22]. Second, switched capacitor filters have to be used together with some other switched filters to convert the sampled output signal into a continuous time signal.

We discuss here briefly the technical necessity and a recent design method for direct simulation of the doubly terminated LC filters by use of capacitances and switches.

1.5.1 Technical necessity

The implementation of resistors in the large scale integration technique is known to be difficult. This stems from their poor temperature and linearity characteristics besides the need for large silicon area to realize them. LSI capacitors, on the contrary, are known to possess low temperature coefficients and a small loss factor. However, the possibility of implementing high precision capacitance ratios (to within 0.1 %) makes it possible to implement LSI filters with natural frequencies dependent only on capacitance ratios [18]. The simplest way may be that of simulating RC active network at first using only capacitances, switches and operational amplifiers; these RC active simulations are used in turn to simulate low sensitivity LC filters (leap frog method). For this purpose, it is necessary to simulate a resistance using capacitances and switches.

In general, one can speak of an apparent resistance whenever energy flows in one direction. An example is the characteristic resistance of a lossless infinite transmission line or the friction in a mechanical circuit. To achieve a similar situation using capacitances and switches, it is sufficient to periodically charge a capacitance from an energy source (current source or a voltage source) and discharge the capacitance partially or totally away from the source. This mechanism - which we used also to design optimum conversion circuits as will be shown later - resembles an apparent resistance to the energy source.

The capacitance C_1 in Fig. 16 is connected to the source E through the switch S until it is charged with a charge Q

$$Q = EC_1.$$

Then it is connected to the input of the operational amplifier where it is completely discharged. If this repeats every T seconds, the resistance seen by the source will be

$$R_1 = \frac{ET}{EC_1} = T/C_1. \quad (1.29)$$

Thus C_1 together with the switch, C_2 and the operational amplifier resemble an integrator with a time constant

$$\tau = R_1 C_2 = T \frac{C_2}{C_1}. \quad (1.30)$$

Using this integrator as a basic building block, the simulation of LC filters is possible. For further analysis and results we refer to [39].

1.5.2 Direct simulation of LC filters

Although it is possible to implement RC circuits through exclusive use of capacitors and switches, the main aim remains always the simulation of LC doubly terminated filters for their excellent sensitivity behavior. The direct solution for such a problem is the use of capacitances, switches and resonant transfer switches to simulate a doubly terminated lossless ladder filter. To allow for an all integrated implementation, the resonant transfer switch is replaced by a "voltage inverter switch" composed of switches, a capacitance and an operational amplifier. The filter can be operated at any sampling rate compatible with the sampling theorem; and there is no restriction to the location of transmission zeros. A distinct advantage is that the number of necessary operational amplifiers needed is substantially lower than for other approaches. For detailed analysis and design we refer to the original publications [21,22].

It is recalled again that the use of switched capacitor filters in signal processing applications necessitates the use of discrete time/continuous time conversion circuitry, whenever the final output signal is needed in a continuous time format.

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Chapter 2: Problem formulation

In this chapter, we present two circuits for the general purpose of converting time discrete signals to continuous signals and for the sampling of continuous signals, together with a complete analysis. The circuits presented have many advantages when compared with other published schemes and analyses methods.

As the circuits suggested belong to the general class of switched circuit, we begin by reviewing shortly some aspects of these. We follow by discussing the general properties of time varying networks.

Section 2.3 is devoted to the presentation of our suggested networks. The pulse impedance theory used to analyse these networks and to interpret the results is presented in Section 2.4. The analysis of the networks suggested then follows in Section 2.5.

2.1 Switched Filters

Circuits containing periodically operated switches have been known long ago as modulators [1], sampling and recovery networks for PAM and PCM [2]; and for many other applications. In general, a switched filter or networks can be of any construction and may contain more than one switch which may be operated simultaneously or at distinct time instants. The analysis of such general networks is normally difficult, and suitable assumptions and restrictions on the circuit construction are necessary to facilitate the study of a certain circuit for a specific application. However, some useful general properties of such circuits were highlighted in many publications [3,4,5,6].

The invention of the resonant transfer principle for lossless bilateral generation of PAM signals created a subclass of switched filters. The main feature of this subclass is that the switch closure time is short relative to the period and the current flowing can then be approximated as a modulated sequence of impulses. Cattermole worked out a method for the analysis of circuits containing periodically operated resonant transfer

switches and gave some design principles [7,8]. However, a general theory was needed to answer the various questions of theoretical and practical importance. This general theory, together with many application possibilities, was presented by Fettweis [9-18].

The conversion networks to be presented in our work make direct use of the aforementioned theory and its related results. Before presenting the networks and their analysis, we discuss first some general properties of periodically time varying networks.

2.2 General properties of periodically varying networks

In order to simplify the subsequent presentation we mention some general properties of periodically varying networks. The discussion holds for linear circuits even if this is not explicitly stated.

2.2.1 Fundamental properties

Since the networks under study can be described by linear differential equations with real coefficients, the analysis by means of complex exponentials remains valid, just as for linear constant networks.

We consider a general network whose parameters are varying with period T , and assume that a complex excitation

$$x(t) = A e^{pt} \quad (2.1)$$

is applied to it, where A and p are complex constants. As was shown by Zadeh [19], the steady-state response $y(t)$ to the excitation (2.1) is of the form

$$y(t) = Y e^{pt} \quad Y = A H(p, t) . \quad (2.2)$$

The function $H(p, t)$, called the system function, is independent of the magnitude of the excitation A , and is periodic in t with period T . $H(p, t)$ is also real for real values of p . Expanding $H(p, t)$ in a Fourier series, we get

$$H(p, t) = \sum_{n=-\infty}^{\infty} H_n e^{jn\Omega t} \quad (2.3)$$

where

$$\Omega = 2\pi F = 2\pi/T. \quad (2.4)$$

The quantity $H_n e^{jn\Omega t}$ is called the component of order n of H . Due to the fact that $H(p,t)$ is real for real values of p , the following relation holds:

$$H_{-n} = H_n^* \quad \text{for } p \text{ real} \quad (2.5)$$

The asterisk means the complex conjugate. Equation (2.1) can be rewritten as

$$y(t) = Y e^{pt} = \sum_{n=-\infty}^{\infty} Y_n e^{(pt+jn\Omega)t} \quad (2.6)$$

From the above we conclude the following:

1. For a periodically-varying circuit, the behaviour is uniquely determined if the system function (2.3) can be determined.
2. For excitations of the form (2.1), the term e^{pt} is just as superfluous as in case of constant circuits.
3. The response (2.6) shows that for $p=j\omega$ a frequency transposition from ω to $n\Omega \pm \omega$ is possible.

2.2.2 Power relations

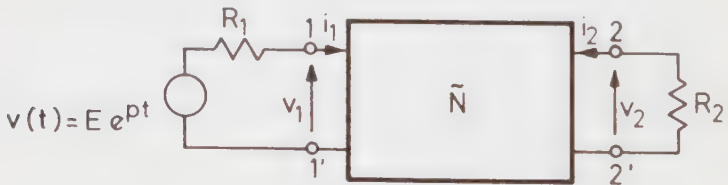


Fig. 2.1 Time-varying network

Consider the circuit of Fig. 2.1 which is assumed to be a linear periodically-varying one. According to the notations used before, we write for the currents and voltages at ports 1 and 2:

$$\begin{aligned} i_1 &= I_1 e^{pt} & v_1 &= V_1 e^{pt} \\ i_2 &= I_2 e^{pt} & v_2 &= V_2 e^{pt} \end{aligned} \quad (2.7)$$

The quantities I_1 , I_2 , V_1 , V_2 are periodic functions which can be expanded in Fourier series, e.g. as

$$I_1 = \sum_{n=-\infty}^{\infty} I_{1n} e^{jn\Omega t} \quad (2.8)$$

Assuming $p=j\omega$, with ω real and

$$\omega \neq k\Omega/2 \quad k = \text{integer} \quad (2.9)$$

the average power flowing across port 1 is of the form

$$P_1 = \overline{v_1 i_1} = \sum_n P_n \quad (2.10)$$

$$P_n = \frac{1}{2} \operatorname{Re} V_{1n}^* I_{1n} \quad (2.11)$$

In studying circuits like that of Fig. 2.1, the following system functions are particularly important:

a) The transfer function S_{21} defined as

$$S_{21}(p, t) = 2\sqrt{R_1/R_2} \frac{V_2}{E} \quad (2.12a)$$

$$= -2\sqrt{R_1 R_2} \frac{I_2}{E} \quad (2.12b)$$

b) The reflection function S_{11} defined as

$$S_{11}(p, t) = (2V_1 - E)/E \quad (2.13a)$$

$$= (E - 2R_1 I_1)/E \quad (2.13b)$$

These definitions are analogous to those of the transfer and reflection coefficients of constant networks, but they differ principally in that they are of periodic nature due to (2.8). Expanding (2.12), (2.13), in a Fourier series and using (2.8) we find:

$$S_{21}(p, t) = \sum_n S_{21n} e^{jn\Omega t} \quad (2.14)$$

with S_{21n} , the conversion coefficient of order n,

$$S_{21n} = 2\sqrt{R_1/R_2} \frac{V_{2n}}{E} \quad (2.15a)$$

$$= -2\sqrt{R_1 R_2} \frac{I_{2n}}{E} \quad (2.15b)$$

Similarly

$$S_{11}(p, t) = \sum_n S_{11n} e^{jn\Omega t} \quad (2.16)$$

with S_{11n} , the reflection coefficient of order n ,

$$S_{110} = (2V_{10} - E)/E = (E - 2I_{10}R_1)/E \quad (2.17a)$$

$$= (Z_{10} - R)/(Z_{10} + R) \quad (2.17b)$$

$$S_{11n} = 2V_{1n}/E = -2I_{1n}R_1/E \quad n \neq 0 \quad (2.18)$$

Z_{10} being the effective input impedance V_{10}/I_{10} at the complex frequency p .

The power transmitted to port 2 is directly related to the conversion function $S_{21}(p, t)$, similarly the reflected power is also related to $S_{11}(p, t)$. Explicit expressions can be given if we notice that the maximum power available from the source is (for $p = j\omega$):

$$P_{\max} = \frac{|E|^2}{8R_1} \quad (2.19)$$

The average power appearing at port 2 is of the form

$$P_2 = \overline{v_2 i_2} = \sum_n P_{2n} \quad (2.20)$$

$$P_{2n} = \frac{1}{2} \operatorname{Re} V_{2n}^* I_{2n}, \quad (2.21)$$

and by using (2.20), (2.21), we get

$$|S_{21n}|^2 = \frac{P_{2n}}{P_{\max}}, \quad (2.22)$$

and

$$\frac{P_2}{P_{\max}} = \sum_n |S_{21n}|^2. \quad (2.23)$$

Furthermore,

$$|S_{110}|^2 + \frac{P_{10}}{P_{\max}} = 1 \quad (2.24)$$

where

$$P_{10} = \frac{1}{2} \operatorname{Re} V_{10}^* I_{10} \quad (2.25)$$

and for $n \neq 0$

$$|S_{11n}|^2 + \frac{P_{1n}}{P_{\max}} = 0. \quad (2.26)$$

To interpret equation (2.26) we notice that the power P_{1n} is negative. This merely means that this power is produced inside the network and is delivered from the network across port 1 to be dissipated in the source resistance R_1 . The total power crossing port 1 is P_1 , where

$$\frac{P_1}{P_{\max}} = \frac{\sum_n P_{1n}}{P_{\max}} = 1 - \sum_n |S_{11n}|^2. \quad (2.27)$$

If the network N were lossless, we conclude directly that $P_1 = P_2$. In other words,

$$\sum_n |S_{11n}|^2 + |S_{21n}|^2 = 1. \quad (2.28)$$

In such a case, the network N is, in general, not allowed to contain resistances or periodically-varying reactances (will usually not be lossless or passive anymore!). Equation (2.28) holds, however, for reactive networks containing periodically operated switches.

2.3 The Conversion Circuits:

As was mentioned before, the function of the circuits under study is either to convert a time discrete signal to a continuous one (interpolation) with shifted frequency equal to $n\Omega \pm \omega$, where n is integer, or to sample a continuous signal efficiently after limiting its bandwidth.

2.3.1 Time-discrete/continuous Signal Conversion [7,8,14,20,21]

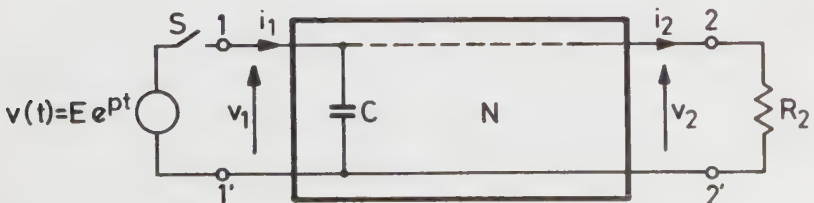


Fig. 2.2 Time-discrete/continuous signal conversion

The suggested circuit is composed of a network N containing constant elements and is terminated at port 2 with a resistance R_2 . At port 1, the network N exhibits an input impedance which reduces at high frequency to a capacitance C ,

$$C = \lim_{p \rightarrow \infty} 1/[pZ(p)] \quad (2.29)$$

The circuit is fed through a switch S from a voltage source with a small internal impedance. The voltage $v(t)$ of the source is proportional (or equivalent) to the periodic, time-discrete signal to be converted. This voltage is assumed to be present for a certain duration T_d every period T (see Fig. 2.3), and it may be e.g. the output of a digital/analog converter at the end of a PCM channel. The switch S is normally open and is closed periodically at the instants t_m , where

$$t_m = mT, \quad (2.30)$$

for a short duration τ which is located inside the interval T_d (Fig. 2.3).

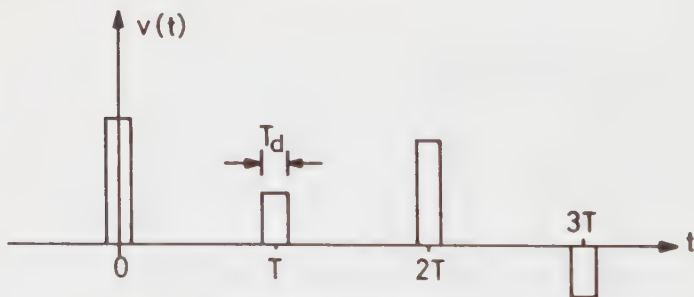


Fig. 2.3 Time-discrete signal

We assume now that the signal $v(t)$ originates from sampling an exponential signal corresponding to a complex frequency p (e. g. $p = j\omega$), so that the values $v(t)$ at the instants of switch closure t_m correspond to

$$v(t_m) = E e^{pt_m} \quad (2.31)$$

The circuit operates in two steps:

a) at the instants t_m given by equation (2.30), the switch closes for the duration τ , assumed ideally to be infinitely short, yet such that the voltage v_1 , across C (and port 1) becomes equal to the source voltage. The charge required to achieve this is carried by the current flowing through the switch S . This process is assumed to take very short time compared to all time constants and resonant periods of the network N , so that we can consider it to be instantaneous.

b) Between consecutive instants mT , the energy accumulated in C is gradually converted into the output signal $v_2 = v_2(t)$. This conversion takes place through the network N , which is assumed to contain only constant elements as mentioned before.

2.3.2 Continuous/discrete time signal conversion (sampling)

In many applications, where a signal is sampled, the sampled values are stored in a capacitance to let sufficient time for e.g. analog to digital conversion for further digital processing or for PCM transmission. The arrangement shown in Fig. 2.4 has been suggested for this purpose [7,8]. The capacitance C_o is charged from the network N by closing the resonant transfer switch, (RS), for a short time once every period. The clamping switch (CS) completely discharges the capacitance C_o each time before RS is again operative. A modification of the circuit makes it possible to replace the resonant-transfer switch RS by e.g. a normal switch and a voltage inverting switch [17] to avoid building the resonant-transfer switch with the use of a coil.

In Section 2.5 both circuits in Fig. 2.2 and 2.4 will undergo a complete analysis to study their behavior in detail. Detailed design algorithm will be given in Chapter 3; but we mention here the advantages of the suggested circuits which may be valid for one or both of them:

1. It is possible to analyse the circuits with the results obtained from the resonant-transfer theory.

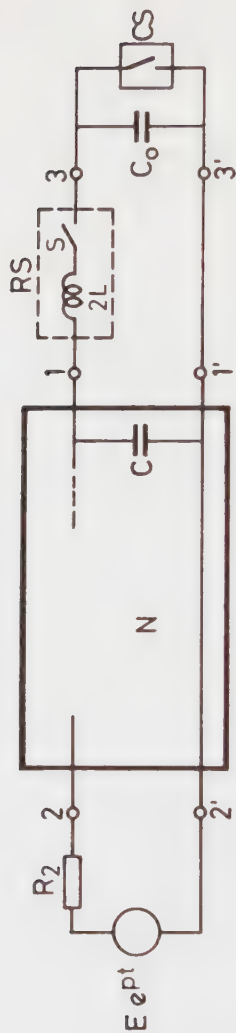


Fig. 2.4 Sampling circuit

2. The circuits are of general nature and may be used for many applications as was mentioned in chapter 1. We stress here the efficiency of the method to yield circuits for direct PCM/FDM conversion.
3. The behavior is described with closed form expressions, giving a possibility to interpret the results easily.
4. The analysis and the design method is independent of a certain network structure. This gives the possibility to realize the results obtained by the most convenient technology; e.g. by using filter structures with minimum number of coils.
5. The circuits will be designed on a maximum power transfer principle. This means optimal sensitivity behavior, immunity to noise, and avoidance of extra amplifiers.
6. The conversion nature of the circuits together with the design on maximum power transfer principle leads to maximal attenuation (suppression) of unwanted sidebands.
7. In multiplexing applications, only one driving stage is required to drive many filters as will be shown in the design examples of chapter 4.
8. The conversion from discrete to continuous signals does not necessitate a correction like that mentioned for circuits described in chapter 1 for PCM recovery (sin x/x correction).

2.4 The Pulse Impedance Theory:

In this section, we present the pulse impedance theory developed originally for application to resonant transfer circuits. The presentation will be kept in the light of our needs to analyse the circuits presented in the last section. For a detailed study of the theory we refer to the original work [9-16].

2.4.1 Definition of Pulse Impedance and Step Resistance:

Consider a passive constant network N with input impedance $Z(p)$ that reduces at high frequency to a capacitance C as was defined by equation (2.29). We assume now that a current i flows into the network. i being a sequence of modulated pulses of the form

$$i = I e^{pt} \qquad I = I(t) = J\Delta(t) \qquad (2.32)$$

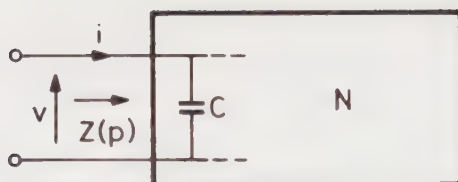


Fig. 2.5

where J is a constant (in general complex) with the dimension of a current. The function $\Delta(t)$ is given by

$$\Delta(t) = T \sum_{m=-\infty}^{\infty} \delta(t-mT) \quad (2.33)$$

In equation (2.33) T is the period of pulse repetition and $\delta(t)$ is a unit impulse. The voltage v appearing across Z , due to the flow of the current i , is then of the form

$$v = V e^{pt} \quad V = J\Phi(p, t) \quad (2.34)$$

$\Phi(p, t)$ is a periodic function in t with period T and is independent of the magnitude of the excitation current J . The voltage v can be determined by noting that the Fourier expansion of (2.33) reveals equal components of the current J , thus

$$I = JT \sum_{m=-\infty}^{\infty} \delta(t-mT) = J \sum_{n=-\infty}^{\infty} e^{jn\Omega t} \quad (2.35)$$

and consequently

$$\Phi(p, t) = \sum_{n=-\infty}^{\infty} Z(p+jn\Omega) e^{jn\Omega t} \quad (2.36)$$

with Ω as defined in equation (2.4).

The voltage v changes suddenly with the arrival of every new current pulse. Designating v_b (and V_b) as the value immediately before and v_a (and V_a) as the value immediately after the arrival of a current pulse at time $t_m=mT$, we get

$$v_a = v_a e^{mpT} \quad v_b = v_b e^{mpT} \quad (2.37)$$

The instants $t_m=mT$ are instants of discontinuity of the voltages

v , V , and the function $\phi(p,t)$. The evaluation of the Fourier series expansion of a function at a point of discontinuity results in the mean value of the function immediately after and immediately before the discontinuity according to the mean value theorem. Using equations (2.36) and (2.37) and the definition

$$U = (V_a + V_b)/2 \quad (2.38)$$

we get

$$U = (V_a + V_b)/2 = J \sum_{n=-\infty}^{\infty} Z(p+jn\Omega) \quad (2.39)$$

The sum in equation (2.39) is called the pulse impedance

$$\tilde{Z} = \sum_{n=-\infty}^{\infty} Z(p+jn\Omega) \quad (2.40)$$

we define furthermore the quantity

$$R_c = (V_a - V_b)/2 J \quad (2.41)$$

and call it the step resistance.

The difference $V_a - V_b$ can be calculated in terms of the input capacitance of the network. A current pulse causes the voltage to change in proportion to the charge carried by this pulse, then

$$(V_a - V_b)C = (V_a - V_b)C e^{jmpT} = JT e^{jmpT} \quad (2.42)$$

giving

$$R_c = T/2C = 1/2 CF \quad (2.43)$$

At this stage, we are able to derive the equivalent circuit of Fig. 2.6a. The importance of this equivalent circuit is not only that it relates the three variables J , U and $\tilde{Z}$ in the simple form

$$U = J \tilde{Z} \quad (2.44)$$

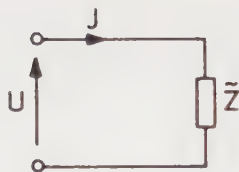


Fig. 2.6a

but it also enables the calculation of the power fed into the network of Fig. 2.5 by the current given by equation (2.32). From equations (2.38-2.41) we can indeed write

$$V_a = (\tilde{Z} + R_C) J \quad (2.45)$$

$$V_b = (\tilde{Z} - R_C) J \quad (2.46)$$

Now, the energy delivered by a single current pulse at the instant t_m , is given by

$$W_m = \frac{1}{2} C (v_a^2 - v_b^2) = T(v_a^2 - v_b^2)/4R_C \quad (2.47)$$

where v_a and v_b are the actual instantaneous values of the voltage across the port 1 of Fig. 2.5. These are given, as usually the case for a complex presentation with $p = j\omega$, by

$$v_a = \operatorname{Re} V_a e^{j\omega t_m} = |V_a| \cos(\omega t_m + \alpha) \quad (2.48a)$$

$$v_b = \operatorname{Re} V_b e^{j\omega t_m} = |V_b| \cos(\omega t_m + \beta) \quad (2.48b)$$

The parameters α, β express the general complex nature of V_a and V_b respectively. Substituting we get

$$W_m = \frac{T}{2R_C} [|V_a|^2 - |V_b|^2 - |V_a|^2 \cos(2\omega t_m + 2\alpha) - |V_b|^2 \cos(2\omega t_m - 2\beta)]. \quad (2.49)$$

Averaging along sufficiently long time, under the condition that

$$\omega \neq k\Omega/2 \quad k = \text{integer} \quad (2.50)$$

we get for the average power P ,

$$P = (|V_a|^2 - |V_b|^2) / 8R_C \quad (2.51)$$

using equations (2.45) and (2.46) we get

$$P = \frac{1}{2} \operatorname{Re} U \cdot J^* = \frac{1}{2} \operatorname{Re} U^* \cdot J \quad (2.52)$$

Equation (2.52) expresses the average power in exactly the same manner as in the case of constant networks. In fact, the equivalent circuit of Fig. 2.6 is suitable for direct calculation of the power as obtained in expression (2.52). Expression (2.52) remains also valid when considering the average power flowing due to the current i when the network N contains energy sources (current or voltage sources). In the circuit of Fig. 2.6b which shows a capacitive input impedance and is fed by a series of current pulses like that of equation (2.32), the average power is still given by equation (2.52) and the equivalent circuit is shown in Fig. 2.6b. The voltage E' is the open circuit voltage measured at port 1.



Fig. 2.6b

Furthermore, if we express the pulse impedance in terms of its real and imaginary parts i.e.,

$$\tilde{Z} = \tilde{R} + j\tilde{X} \quad (2.53)$$

Then, the average power is given by (when N does not contain energy sources)

$$P = \frac{1}{2} \tilde{R} |J|^2 \quad (2.54)$$

By noting that P must be positive as a result of the passivity of the network N in Fig. 2.5, we get

$$\tilde{R} \geq 0 \quad (2.55)$$

This same result is obtained if we notice that

$$\tilde{R} = \sum_{n=-\infty}^{\infty} R(j\omega + jn\Omega) \quad (2.56)$$

$$\tilde{X} = \sum_{n=-\infty}^{\infty} X(j\omega + jn\Omega) \quad (2.57)$$

where $R(j\omega)$ and $X(j\omega)$ are given by

$$Z(j\omega) = R(j\omega) + j X(j\omega) \quad (2.58)$$

The expressions (2.40), (2.56) and (2.57) were obtained on the basis of a frequency-domain analysis. Equally well, other usefull expressions can be obtained from analysis in the time domain. If we write the impedance $Z(p)$ in a partial fraction form*

$$Z(p) = \sum_{i=1}^k \frac{B_i}{p - p_i} \quad (2.59)$$

k being the order of Z , and define $A(t)$ as the impulse response of $Z(p)$, i.e. $Z(p)$ is the Laplace transform of $A(t)$;

$$Z(p) = \mathcal{L}\{A(t)\} \quad (2.60)$$

we get then the following two important alternative expressions for the pulse impedance

$$\tilde{Z} = R_C + \sum_{m=1}^{\infty} e^{-mpT} A(mT) \quad (2.61)$$

and

$$\tilde{Z} = T/2 \sum_{i=1}^N \frac{B_i (1 - \psi \psi_i)}{\psi - \psi_i} \quad (2.62)$$

with

$$\psi = \tanh(pT/2) \quad (2.63)$$

$$\psi_i = \tanh(p_i T/2) \quad (2.64)$$

* We consider here the common practical case where $Z(p)$ has only simple poles but a similar analysis holds for the general case of multiple poles.

From (2.59) and (2.29) we get also

$$C = \lim_{p \rightarrow \infty} pZ(p) = \sum_{i=1}^k B_i \quad (2.65)$$

In equation (2.62) the pulse impedance $\tilde{Z}$ is expressed as a rational function in the variable ψ . This leads to several interesting properties of Z as well as of the transformation (2.63). Before discussing these properties, we introduce the normalized quantities which we are going to use for our further analysis. As a normalizing quantity we define the resistance R_0 which can be specified in each individual case.

$$Z = r + jx = Z/R_0, \quad r = R/R_0, \quad x = X/R_0 \quad (2.66)$$

$$\tilde{Z} = \tilde{r} + j\tilde{x} = \tilde{Z}/R_0, \quad \tilde{r} = \tilde{R}/R_0, \quad \tilde{x} = \tilde{X}/R_0 \quad (2.67)$$

$$C_0 = T/2R_0, \quad c = C/C_0 = R_0/R_c \quad (2.68)$$

$$\lambda = pT/2, \quad \lambda_i = p_i T/2 \quad (2.69)$$

$$v = \omega T/2 = \pi f/F \quad (2.70)$$

$$b_i = B_i C_0 \quad (2.71)$$

and accordingly

$$\psi = \tanh \lambda \quad (2.72)$$

$$\phi = \tan v \quad (2.73)$$

$$Z(\lambda) = \sum_{i=1}^k \frac{b_i}{\lambda - \lambda_i} \quad (2.74)$$

$$\frac{1}{c} = \lim_{\lambda \rightarrow \infty} \lambda Z(\lambda) = \sum_{i=1}^N b_i \quad (2.75)$$

$$z(\psi) = \sum_{i=1}^k b_i \frac{1 - \phi \phi_i}{\psi - \psi_i}, \quad (2.76a)$$

and for the current transfer function we can use the notation $m(\lambda)$ where

$$m(\lambda) = M(p) \quad (2.76b)$$

2.4.2 Properties of the ψ transformation, the pulse impedance and the step resistance:

The independent variable z used in the Z -transform theory is related to the complex frequency variable p (and λ) by

$$z = e^{pT} = e^{2\lambda} \quad (2.77)$$

using equation (2.63) or (2.72) we get

$$\psi = \frac{z-1}{z+1} \quad (2.78)$$

The variable ψ has the interesting property

$$\operatorname{Re} \psi \begin{matrix} > \\ < \end{matrix} 0 \quad \text{if} \quad \operatorname{Re} p \begin{matrix} > \\ < \end{matrix} 0 \quad (2.79)$$

where equally placed symbols correspond to one another.

The pulse impedance $\tilde{Z}$ (and $\tilde{z}$) is a rational function in the variable ψ and can be considered as the ψ -transform of the impedance $Z(p)$. Although $\tilde{Z}(\psi)$ is unique for a given $Z(p)$, there exists an infinite number of inverse transforms $Z(p)$ for a given $\tilde{Z}(\psi)$. This follows from the fact that there exists an infinite number of different p_i which satisfy equation (2.64) for a given ψ_i .

Throughout the following discussions, it is necessary to express the dependence of $\tilde{Z}$ upon the independent variables ψ or p . To avoid confusion, the $\tilde{Z}(\psi)$ will mean the dependence of $\tilde{Z}$ upon ψ in the sense of equation (2.62). Whenever it is necessary to manifest the dependence of pulse impedance $\tilde{Z}$ upon the complex frequency p or upon $j\omega$ we would write $\tilde{Z}(p)$ or $\tilde{Z}(j\omega)$; and it must be understood that this is not a mere replacement of ψ by p or $j\omega$, but the dependence of $\tilde{Z}$ on p or $j\omega$ in the sense of equations (2.40), (2.56), (2.57) and (2.53).

From the previous discussion we can state the following properties for $\tilde{Z}$ [9,10]:

1. $\tilde{Z}(\psi)$ is a Brune function (rational positive real function) of the variable ψ if $Z(p)$ is a Brune function of the variable p .

The converse, however, is not always true.

2. $\tilde{Z}(\phi)$ is minimum reactive in ϕ if and only if $Z(p)$ is minimum reactive in p .
3. If, for $\phi = j\omega$, $\tilde{R}(\omega)$ is zero for any particular value of ω , it is identically zero for all ω [this follows from equation (2.56) by noting that $R(\omega)$ cannot be zero in an infinite number of points unless it is identically zero for all ω].
4. $\tilde{Z}(\phi)$ is a reactance function of the variable ϕ if $Z(p)$ is a reactance function of the variable p , and vice versa.
5. $\tilde{Z}(\phi)$ and R_C are related by

$$\tilde{Z}(1) = R_C, \quad \tilde{Z}(-1) = -R_C \quad (2.80)$$

6. $\tilde{Z}(\omega)$ is the input impedance of an ideal open circuit filter in the ϕ domain if $Z(p)$ is the input impedance of an ideal open circuit filter in the p domain. This point will be discussed in some detail later in this chapter.
7. $\tilde{Z}$ is periodic in p in the imaginary direction, with a period $j\Omega$, i.e.

$$\tilde{Z}(p) = \tilde{Z}(p + jn\Omega) \quad n \text{ integer} \quad (2.81)$$

8. $\tilde{R}(\omega)$ is an even function of ω , while $\tilde{X}(\omega)$ is an odd one. This yields

$$\tilde{Z}(j\omega) = \tilde{Z}^*(-j\omega) \quad (2.82)$$

9. At $\omega = k\Omega/2$, k an odd integer, $\tilde{X}$ is zero (except under special conditions in case of a non minimum reactive $Z(p)$) and $\tilde{R}$ has an extremum (either maximum or minimum).

2.5 Analysis of the conversion circuits:

2.5.1 Time-discrete/continuous signal conversion circuit

In this section, the circuits introduced in section 2.3 are analysed. As we are interested in the conversion behavior, we have only to determine the transfer function given by equation (2.12). We stress here that the analysis involves not only voltage relations, as is usually done in the literature, but also power aspects.

In the circuit of Fig. 2.2, the current transfer function M is defined as

$$M(j\omega) = \frac{I_2(j\omega)}{I_1(j\omega)} \quad (2.83)$$

when the network N is terminated by the resistance R_2 at port 2. The input impedance Z satisfies equation (2.29) and we can consider the current i_1 as a series of pulses as in equation (2.32). The switch closure, at $t = t_m$, causes the flow of a current pulse with the result that the voltage v_1 reaches the value [see equation (2.31)]

$$v_{1a}(t_m) = E e^{pt_m} \quad (2.84)$$

According to the definitions (2.37) we have

$$V_{1a} = E \quad (2.85)$$

and the intensity of the current pulses, J , can now be determined from equation (2.45) as

$$V_{1a} = (\tilde{Z} + R_C)J = E \quad (2.86)$$

$$J = E/(\tilde{Z} + R_C) \quad (2.87)$$

The input current i_1 is now given by

$$i_1 = I_1 e^{pt} = J\Delta(t) e^{pt} \quad (2.88)$$

with J as given by equation (2.87). Equation (2.86) suggests the equivalent circuit of Fig. 2.7a.

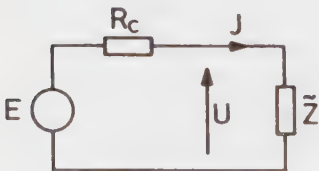


Fig. 2.7a

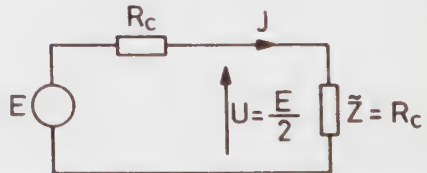


Fig. 2.7b

The output voltage v_2 can be now determined as the summation of the different (Fourier) components of i_2 multiplied by R_2 , and noting that I_1 has equal components, J ,

$$v_2(t) = V_2(t) e^{pt} \quad V_2(t) = \sum_{n=-\infty}^{\infty} V_{2n} e^{jn\Omega t} \quad (2.89)$$

with $V_{2n} = I_{2n} R_2$

$$= J R_2 M(p+jn\Omega) \quad (2.90)$$

This helps now the calculation of the conversion coefficient of order n in a similar manner as in equation (2.12),

$$S_n = S_{21n} = 2 \sqrt{\frac{R_C}{R_2}} \frac{V_{2n}}{E} \quad (2.91)$$

the constant factor $2 \sqrt{R_C/R_2}$ having been chosen in such away that for $p = j\omega$

$$|S_n|^2 = \frac{P_{2n}}{P_{\max}} \quad (2.92)$$

P_{2n} is the power delivered to R_2 by the component of order n at the frequency $\omega+n\Omega$, i.e.

$$P_{2n} = \frac{|V_{2n}|^2}{2R_2} \quad (2.93)$$

$P_{\max}$ is the maximum available power, which can be delivered to the network N through the switch S and can be calculated from Fig. 2.7a, thus

$$P_{\max} = \frac{|E|^2}{8R_C} \quad (2.94)$$

This means: if the network N can be chosen to satisfy the condition

$$\tilde{Z} = R_C \quad (2.95)$$

then it will consume the maximum power (2.94) from the source. This is shown schematically in Fig. 2.7b.

From equation (2.88-2.92) we can now write

$$S_n = \frac{2\sqrt{R_C R_2} M(p+jn\Omega)}{\tilde{Z} + R_C}, \quad (2.96a)$$

and in terms of normalized quantities

$$S_n = \frac{2\sqrt{r_c r_2} m(\lambda+jn\pi)}{\tilde{z} + r_c} \quad (2.96b)$$

Thus, it is possible to describe the conversion behavior in the closed form expression (2.96). In investigating solution possibilities for an optimum conversion network, the expression (2.96) helps to prove the existence of an ideal theoretical solution and facilitates the design of practical optimally dimensioned circuits, as will be shown later.

2.5.2 Sampling circuit:

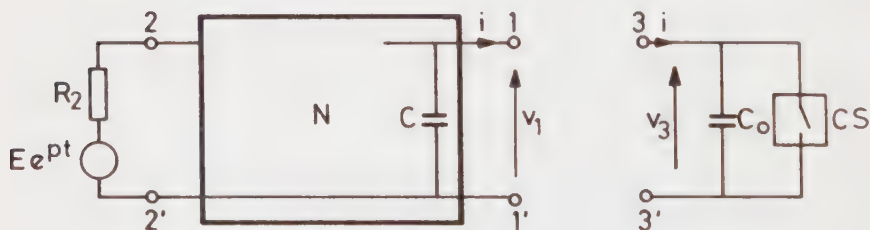


Fig. 2.8

The circuit of Fig. 2.4 is drawn again in Fig. 2.8 after dividing it in two parts by eliminating the resonant transfer switch (RS). The current flowing through (RS) is of pulsive form and can be described by equation (2.32) as long as the long time

behavior is considered, i.e. the behavior of the circuit during the time between the arrival of the successive pulses. The pulse impedance seen at port 3 of Fig. (2.8b) can be determined by noting that the arrival of a current pulse causes V_3 to change from zero ($V_{3b} = 0$, complete discharge of C_o) to a value V_{3a} with

$$(V_{3a} - V_{3b})/2 = J R_{C_o} \quad , \quad R_{C_o} = T/2C_o$$

$$\text{i.e.,} \quad V_{3a} = 2 R_{C_o} J \quad (2.97)$$

giving

$$U_3 = (V_{3a} + V_{3b})/2 = J R_{C_o} \quad (2.98)$$

$$\tilde{Z}_3 = \frac{U_3}{J} = T/2C_o = R_{C_o} \quad (2.99)$$

At port 1, the voltage v_1 is a result of the presence of the source $E e^{pt}$ and the flow of the current i . By applying the superposition theorem, we get for the voltage v_1

$$v_1 = E_o - \tilde{Z} J \quad (2.100)$$

E_o is the voltage that would appear across 1-1' if the voltage source were acting alone (and dropping the term e^{pt}), while $\tilde{Z}$ is the pulse impedance of the network N when terminated by R_2 at port 2.

To relate now the conditions at port 1 with those at port 3 we have to study also the short time behavior of the resonant transfer switch RS. For this purpose the switch and the two capacitances C and C_o are redrawn in Fig. 2.9.

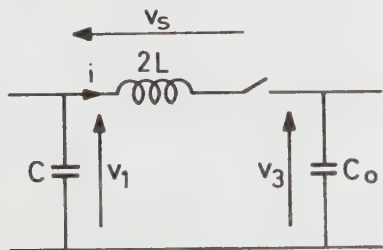


Fig. 2.9

As was described in section 1.2 , the voltages v_1 and v_3 change due to the flow of the current i . For the duration during which the switch is closed we can write

$$i = \frac{v_{1b} - v_{3b}}{2\omega_o L} \sin \omega_o t = \frac{v_{sb}}{2\omega_o L} \sin \omega_o t \quad (2.101)$$

$$v_1 = \frac{C}{C_o + C} v_{1b} + \frac{C_o}{C + C_o} v_{3b} + \frac{C_o}{C + C_o} (v_{1b} - v_{3b}) \cos \omega_o t \quad (2.102)$$

$$v_3 = \frac{C}{C_o + C} v_{1b} + \frac{C_o}{C + C_o} v_{3b} + \frac{C}{C + C_o} (v_{3b} - v_{1b}) \cos \omega_o t \quad (2.103)$$

where

$$\omega_o = \frac{1}{\sqrt{2L \frac{CC_o}{C + C_o}}} \quad (2.104a)$$

and we then have

$$v_s = v_1 - v_3 = (v_{1b} - v_{3b}) \cos \omega_o t, \quad (2.104b)$$

with t now measured from the instant when the switch is closed; v_{1b} and v_{3b} are the values of the voltages v_1 and v_3 immediately before switch closure. After a time τ , the switch is opened again and,

$$\tau = \frac{\pi}{\omega_o} \quad (2.105)$$

ensures that the current then flowing is equal to zero

$$i(\tau) = 0 \quad (2.106)$$

$$v_1(\tau) = v_{1a} = \frac{C - C_o}{C_o + C} v_{1b} + \frac{2C_o}{C + C_o} v_{3b} \quad (2.107)$$

$$v_3(\tau) = v_{3a} = \frac{2C}{C + C_o} v_{1b} + \frac{C_o - C}{C + C_o} v_{3b} \quad (2.108)$$

$$U_1 = (v_{1a} + v_{1b})/2 = \frac{1}{e^{pt_m}} (v_{1a} + v_{1b}) \quad (2.109)$$

$$U_3 = (V_{3a} + V_{3b})/2 = \frac{1}{pt_m} (v_{3a} + v_{3b}) \quad (2.110)$$

giving the wanted relation between U_1 and U_3

$$U_1 = U_3 = \frac{2}{C+C_o} (C V_{1b} + C_o V_{3b}) \quad (2.111)$$

Using equations (2.98), (2.99), (2.100) and (2.111) and the fact that the same current i flows through ports 1, 3 we can construct the equivalent circuit of Fig. 2.10. To calculate a transfer func-

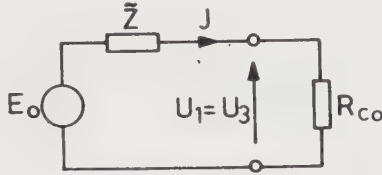


Fig. 2.10

tion for the sampling circuit, we define,

$$S = k_S \frac{V_{3a}}{E} \quad (2.112)$$

and we have now to choose the constant k_S so that

$$|S(j\omega)|^2 = \frac{P_3}{P_{\max}} \quad (2.113)$$

with

$$P_{\max} = \frac{|E|^2}{8R_2} \quad (2.114)$$

is the maximum available power from the source at port 2 and P_3 is the average power delivered to C_o through port 3, i.e. through the resonant transfer switch (RS). From Fig. 2.10 we can write

$$P_3 = \frac{|U_3|^2}{2R_{C_o}} = \frac{|V_{3a}|^2}{8R_{C_o}} \quad (2.115)$$

From equations (2.110)-(2.113) we write now for S

$$S = \sqrt{\frac{R_2}{R_C}} \frac{V_{3a}}{E} \quad (2.116)$$

As the network N is assumed to be reciprocal, the current transfer ratio as defined by equation (2.83) in the direction $1 \rightarrow 2$ is equal to the open circuit voltage ratio in the direction $2 \rightarrow 1$ when the network N is terminated in both cases at port 2 by R_2 . Thus

$$\frac{E_o}{E} = M \quad (2.117)$$

and we get from (2.96), (2.116) and (2.117)

$$S = 2 \sqrt{R_2 R_{C_o}} M(p) / (\tilde{Z} + R_{C_o}) \quad (2.118)$$

A comparison of the two expressions (2.118) and (2.96) (for the case $n=0$) shows that a proper choice for the network N to achieve optimum behavior of the conversion function can be automatically used to design an optimum sampling network if the capacitance C_o is chosen such that

$$C_o = C \quad (2.119)$$

is satisfied. As a result, in our search for an optimum construction for the network N in both cases of Fig. 2.2 and 2.4 it is only sufficient to consider the first case only.

2.6 Design of ideal conversion filters:

We consider the theoretical aspects of designing the (filter) network N for ideal time-discrete/continuous signal conversion. It is principally possible to convert a discrete time signal into a double-side-band continuous signal around a carrier [9,10], but we limit the discussion to the more practically important case of single-side-band. For further discussion, the following definitions are useful:

The set PB: is the set containing all frequencies ω and the normalized frequencies $\nu = \omega T/2$ located in the passband of a filter or conversion circuit.

The set SB: is the set containing all frequencies ω and the normalized frequencies $\nu = \omega T/2$ located in the stopband of a filter or conversion circuit.

2.6.1 The Ideal Universal Filter

The general form of the conversion coefficient of order n was given in equation (2.96). In particular, when it is required that the original signal be recovered, the conversion coefficient of order zero is considered

$$S_0 = \frac{2 \sqrt{R_C R_2} M(p)}{\bar{Z} + R_C} \quad (2.120a)$$

$$= \frac{2 \sqrt{r_C r_2} m(\lambda)}{\bar{z} + r_C} \quad (2.120b)$$

In order to achieve perfect conversion, the conversion coefficient S_n is required to attain its highest possible value, namely 1, in the required band. From equations (2.96) this necessitates that the parameters of the network N have to satisfy:

$$\bar{Z}(j\omega) = R(j\omega) = R_0 \quad \omega \in PB \quad (2.121)$$

$$|M(j\omega)|^2 = R_0/R_2 \quad \omega \in PB \quad (2.122)$$

where R_0 is a constant, and condition (2.122) is expressing the losslessness of N . Another condition is still necessary, namely, that N must show an impedance Z_1 seen at port 1 which reduces to a capacitance C at high frequencies according to equation (2.29); furthermore, it is required that

$$R_C = R_0 = T/2C \quad (2.123)$$

It was proven that networks which satisfy equations (2.121) and (2.122), satisfy automatically condition (2.123), and are called ideal universal filters [9,10]. The design of the network N as an ideal universal filter guarantees perfect conversion of the discrete-time signal into a continuous one in the required band and complete suppression of all other sidebands. Conditions (2.121), (2.122) and (2.123) can be expressed in terms of the normalized quantities. Choosing R_0 as the normalizing resistance, we get

$$\bar{z}(j\phi) = 1 \quad (2.124a)$$

$$r(jv) = 1 \quad (2.124b)$$

$$m(jv) = 1/r_2 \quad (2.124c)$$

$$r_C = 1 \quad (2.124d)$$

for $v \in PB$.

The ideal universal filter was originally invented for applications in PAM exchanges based on the resonant-transfer principle. Although condition (2.124a) is expressed in terms of the pulse impedance while conditions (2.124b) and (2.124c) in terms of the real part of the input impedance and the current transfer ratio, the proof of existence of the ideal universal filter is straight forward. The following presentation is limited to the case of lowpass filters ($n=0$), but can be extended to other cases without major difficulty.

2.6.2 The ideal open circuit filter:

A two-port network N , terminated at port 2 by a resistance R_2 is said to be an ideal open circuit filter when the real part of the input impedance R satisfies

$$\begin{aligned} r(v) = \frac{R}{R_0} &= 1 & v \in PB \\ &= 0 & v \in SB \end{aligned} \quad (2.124)$$

R_0 is a constant and the passband extends from zero to the cut-off frequency ω_C . Under the condition that the network N is minimum reactive, the imaginary part $x(v)$ corresponding to the real part (2.124) can be determined;

$$x(v) = -j \frac{1}{\pi} \ln \left| \frac{v+v_C}{v-v_C} \right| \quad (2.125)$$

with v_C satisfying

$$v_C = \omega_C T / 2 = \pi f_C T \quad (2.126)$$

With the use of equation (2.56) we can calculate the real part of $\tilde{z}$,

$$\tilde{r}(\phi) = \begin{cases} 1 & |\phi| < \phi_C \\ 0 & |\phi| > \phi_C \end{cases} \quad (2.127)$$

As $\tilde{z}$ is minimum reactive when z is also minimum reactive, the imaginary part $\tilde{x}(\phi)$ corresponding to $\tilde{r}(\phi)$ is now

$$\tilde{x}(\phi) = -j \frac{1}{\pi} \ln \left| \frac{\phi + \phi_C}{\phi - \phi_C} \right| \quad (2.128)$$

In other words, the pulse impedance $\tilde{z}$ is the input impedance of an ideal open circuit filter in the ϕ -domain. For the choice of the cutoff frequency such that

$$f_C = F/2 = \frac{1}{2T} \quad (2.129)$$

the ideal open circuit filter becomes a special case of the general ideal universal filter, since

$$\phi_C = \infty \quad (2.130)$$

$$\text{giving } \tilde{x}(\phi) = 0 \quad (2.131)$$

$$\text{and } \tilde{z}(\phi) = 1 \quad (2.132)$$

2.6.3 Compensation:

If condition (2.129) does not hold, i.e. if

$$f_C < F/2 \quad (2.133)$$

it is still possible to build an ideal universal filter by compensating an ideal open-circuit filter as shown in Fig. 2.11. The network N comprises now the ideal open circuit filter N' and the network A used for compensation. The ideal open-circuit filter has a cutoff frequency $f_C < F/2$ and its normalized input impedance $\tilde{z}'$ satisfy equations (2.124)-(2.128). The network A has to compensate for the imaginary part $\tilde{x}'(\phi)$, i.e.,

$$\tilde{z}'A = j \frac{1}{\pi} \ln \left| \frac{\phi_C + \phi}{\phi_C - \phi} \right| \quad \begin{array}{l} |\phi| < \phi_C \\ \text{arbitrary} \quad |\phi| > \phi_C \end{array} \quad (2.134)$$

One possibility to realize the network A is the use of an ideal open-circuit bandpass filter with cutoff frequencies f_C and $F/2$.

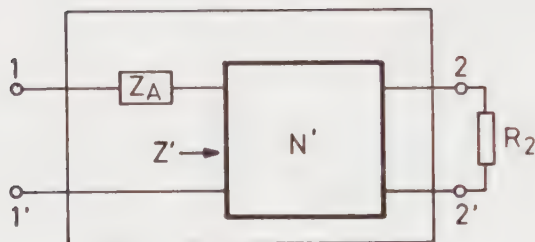


Fig. 2.11 Design of ideal-universal-filter by compensation

However, the design of A as a pure reactive network was seen to be possible [9 (Chapter 6), 10].

The use of the above mentioned results in designing practical circuits necessitates the approximation of an ideal open circuit filter (see e.g. [22]), and the use of a simple compensating network e.g. a parallel resonant circuit. Although, in principle, this method allows the design of arbitrary good filters, it does not furnish filters that are optimum for a given performance criterion, such as the case of the smallest number of elements or smallest number of inductances for a given attenuation requirement. In chapter 3, we present a complete method for designing optimum conversion networks. The networks are optimum in the sense that for a given number of elements, the best conversion behavior is reached besides the fact that the overall behavior exhibits low sensitivity to variations in element values.

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Chapter 3

Design of Optimum Conversion Circuits

With the help of a parametric representation of Brune functions [1], a realizability theorem is presented to prove the possibility of building networks with a finite number of parameters (and consequently with a finite number of elements) as conversion circuits of the type presented in Chapter 2. The design of the optimum filter reduces then to a mathematical optimization problem. A suitable solution algorithm is presented which gives the optimum values of network parameters. The calculation of the element values then follows by the use of available synthesis programs [2].

We begin by presenting a comparison of the conversion circuits with doubly-terminated filter circuits and studying the sensitivity behavior of circuits designed on the maximum power transfer principle.

3.1 Comparison with constant filter networks:

3.1.1 Properties of constant doubly-terminated filter networks

We discuss shortly the principal properties of lossless passive filters designed on the insertion-loss principle. We consider here the circuit of Fig. 3.1, and

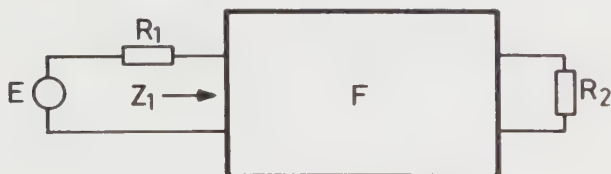


Fig. 3.1 Doubly-terminated filter

define the transfer function as usual by

$$T(p) = 2\sqrt{\frac{R_1}{R_2}} \frac{V_2}{E} \quad (3.1)$$

which expresses the transmission from port 1 to port 2. The loss

A in Nepers and the phase B in radians are related to T by

$$A + jB = -\ln T(j\omega) \quad (3.2)$$

For design purposes, the transfer function is normally expressed as a rational function,

$$T(p) = \frac{f(p)}{g(p)} \quad (3.3)$$

For reciprocal circuits, the polynomial $f(p)$ is an odd or an even polynomial in p , of degree equal to or less than the degree of the polynomial $g(p)$. The polynomial $g(p)$, expressing the natural frequencies of the circuit, must be a Hurwitz polynomial with zeros lying in the left-hand side of the complex p -plane and appearing in the form of complex conjugate pairs or as simple zeros on the negative real axis.

Under the restrictions mentioned above for $f(p)$ and $g(p)$, the proper choice of the polynomials has to be made such that well-defined specifications on the attenuations A or the phase B or on both have to be strictly met. Typical requirements on the attenuation A for a low pass filter are shown in Fig. 3.2a. These specifications can be translated to specifications on the transfer function $|T(j\omega)|$ as shown in Fig. 3.2b, with

$$T_d = e^{-A_d} \quad (3.4)$$

$$T_s = e^{-A_s} \quad (3.5)$$

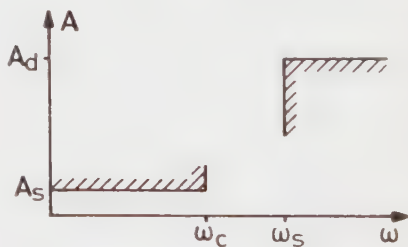


Fig. 3.2a

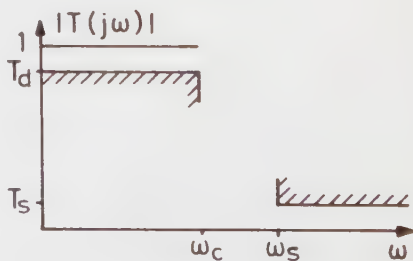


Fig. 3.2b

Now, the order of the polynomial $g(p)$ - which is at the same time the filter order - and the zeros of $g(p)$ and $f(p)$ have to be determined. The determination of the suitable polynomials is made usually numerically but for special groups of filter circuits it is possible to derive explicit formulae for the zeros, such as for Butterworth or Chebychew filters.

Parallel to the transfer function $T(p)$, there is always the reflection function $\rho_1(p)$. The reflection $\rho_1(p)$ at port 1 is expressed in terms of the input impedance Z_1 as

$$\rho_1(p) = \frac{Z_1(p) - R_1}{Z_1(p) + R_1} \quad (3.6)$$

which can be also expressed in the form of a rational function

$$\rho_1(p) = \frac{h(p)}{g(p)} \quad (3.7)$$

where the zeros of $h(p)$ are required to be either real or conjugate complex and can lie anywhere in the p -plane. For lossless filter networks, the following power relation (Feldtkeller equation) holds:

$$1 = |\rho_1(j\omega)|^2 + |T(j\omega)|^2 \quad (3.8)$$

expressing the conservation of energy in the circuit, and leading to:

$$g(p) g_*(p) = h(p) h_*(p) + f(p) f_*(p) \quad (3.9)$$

where the asterisk designates the Hurwitz conjugate, as e.g.

$$g_*(p) = g(-p) \quad \text{etc.} \quad (3.10)$$

In realizing a circuit to meet certain transmission requirements, it is assumed that $h(p)$ and $f(p)$ are known or can be determined. The determination of $g(p)$ follows by solving equation (3.9) and proper choice of the roots of $g(p)$. However, in some cases one can start by determining $f(p)$ and $g(p)$ and then solve for $h(p)$. As it can be seen, there are, in general, several choices for $h(p)$ leading to different circuits and different values of Z_1

(except for the case where all the zeros of $h(p)$ lie on the $j\omega$ axis). This point is very important for our considerations when designing conversion circuits because of the role played by the input capacitance.

Design steps for a given combination of $g(p)$, $f(p)$ and $h(p)$ are well known and need not be discussed here [2,3]. But we mention here that designs based on the maximum power transfer in the passband on the one hand and ladder realizations on the other hand result in structures with minimum sensitivity of the overall attenuation behavior with respect to variations in the element values [4,5]. The property of minimum sensitivity in the passband is evident from the fact that

$$|T(j\omega)| \leq 1 \quad (3.11)$$

$$\text{and} \quad A \geq 0 \quad (3.12)$$

so that for circuits designed on maximum power transfer principle, it is possible to keep $|T(j\omega)|$ very near to 1 (A very near to zero) in the entire passband and even attain the value 1 at several frequency points $\omega_1, \omega_2, \dots$, in the passband as is schematically shown in Fig. 3.3. We assume that the arbitrary element C_m of the

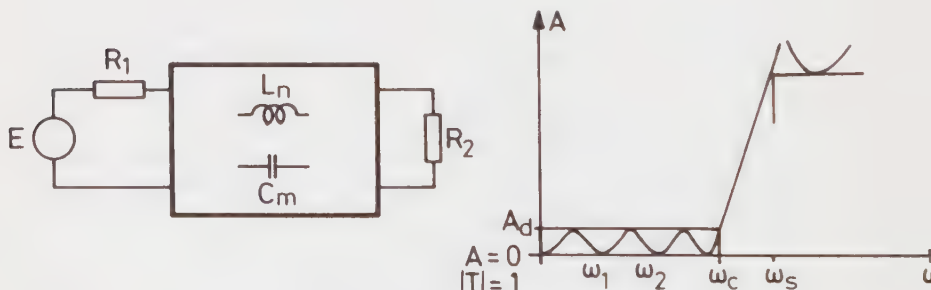


Fig. 3.3

filter is varied around its nominal value C_m^0 ; the corresponding variation in the attenuation is schematically drawn at the frequency ω_1 in Fig. 3.4 and we can write

$$\left. \frac{\partial A}{\partial C_m} \right|_{\substack{C_m = C_m^0 \\ \omega = \omega_1}} = 0 \quad (3.13)$$

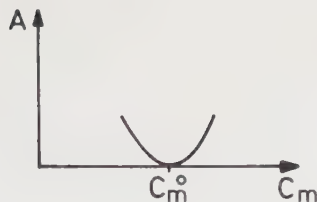


Fig. 3.4

It is expected that away from the point ω_1 , the sensitivity is kept small when the maximum attenuation in the passband, A_d , is kept sufficiently small. This property is not necessarily restricted to reactance two-port networks, it remains also valid for any network provided it operates to give maximum transfer of power.

When the filter is realized in a ladder form, it is possible to realize transmission zeros only if a shunt admittance or a series impedance has a pole. As a consequence, the branches are responsible for creating the transmission zeros. This makes the tuning of the ladder filter relatively simple and keeps the sensitivity of the transmission zero locations and the attenuation in the stopband relatively small as compared with the case in other structures where the realization of transmission zeros depends on a bridge-type balance of several branch impedances.

We finally write a third expression for the transfer function of the circuit of Fig. 3.1 in terms of the input impedance Z_1 and the current transfer ratio M . The transfer function $T(p)$ is then

$$T(p) = 2 \sqrt{R_1 R_2} \frac{M(p)}{R_1 + Z_1} \quad (3.14)$$

If a lossless filter is designed to exhibit a small loss in the passband and sufficiently high attenuation in the stopband and if both terminating resistances R_1 and R_2 are set equal to 1, we can write the following relations:

$$R_1 = R_2 = 1 \quad (3.15)$$

$$T(p) = 2 \frac{M(p)}{1 + Z_1(p)} \quad (3.16)$$

$$T(j\omega) \approx 1 \quad \text{in the passband} \quad (3.17)$$

$$T(j\omega) \approx 0 \quad \text{in the stopband} \quad (3.18)$$

$$|M(j\omega)|^2 = \operatorname{Re} Z_1(j\omega) \quad (3.19)$$

It is observed that for such a filter the input impedance Z_1 is almost real in the passband and is near the value 1, while in the stopband it is almost purely imaginary. From these properties and with the aid of equations (3.16) and (3.19) the filtering behavior as expressed by equations (3.17) and (3.18) can be explained.

3.1.2 Comparison between doubly-terminated filter networks and conversion circuits

1. It was seen that a single time-independent expression can describe the behavior of a doubly-terminated filter over the entire complex-plane, while for conversion circuits (and periodically-varying networks in general), either a single time-dependent transfer function (see equation 2.12) or an infinite number of conversion coefficients (see equations 2.15 and 2.96) are necessary to describe the conversion behavior of the circuit. However, for the conversion circuits under consideration, we can write

$$S_0(p + jn\Omega) = 2 \sqrt{R_C R_2} \frac{M(p + jn\Omega)}{R_C + Z(p + jn\Omega)} \quad (3.20)$$

$$= 2 \sqrt{R_C R_2} \frac{M(p + jn\Omega)}{R_C + Z(p)} \quad (3.21)$$

$$= S_n(p) \quad (3.22)$$

so that all conversion coefficients can be expressed in terms of S_0 and it is sufficient to consider only S_0 to describe the whole behavior of the conversion circuit at all frequencies. In other words, the suppression of unwanted components at frequencies $\omega + n\Omega$ is equally obtained by designs which keep S_0 small at the frequencies $\omega + n\Omega$. For the attenuation of the conversion circuit we define

$$A = - \ln |S_o(j\omega)| \quad \text{Neper} \quad (3.23)$$

$$\text{or} \quad A = - 20 \lg_{10} |S_o(j\omega)| \quad \text{dB} \quad (3.24)$$

2. It is always possible to design doubly-terminated filter networks of any order with arbitrary small attenuation in the passband; and the attenuation may reach the value zero at certain frequency points inside the passband. For conversion circuits with finite number of elements the situation is different. The total output power P_2 appearing at port 2 is the sum of the powers P_{2n} at the frequencies $\omega + n\Omega$, where ω is the frequency of the source signal, thus

$$P_2 = \sum_{n=-\infty}^{\infty} P_{2n} \quad (3.25)$$

It is thus impossible that the attenuation can reach the value zero anywhere inside the passband. This is because a complete transmission at a certain frequency ω_1 , located in the passband of the conversion circuit, would necessitate that the network N must have an infinite number of attenuation poles (transmission zeros) at the frequencies

$$\omega_1 + n\Omega, \quad n = \dots, -3, -2, -1, 1, 2, 3, \dots \quad (3.26)$$

which is inconsistent with the assumption that N contains a finite number of elements.

Moreover; the choice of the highest attenuation in the passband (A_d) is not arbitrary. We illustrate this by means of the following example. If we require that the attenuation inside the passband of a certain conversion circuit does not exceed 0.1 dB, this amounts to requiring that

$$1 > |S_o(j\omega)|^2 \geq 0.97723 \quad \omega \in \text{PB} \quad (3.27)$$

The remaining power must then either be reflected at the input or appear at the output at the frequencies $\omega + n\Omega$; thus

$$|S_o(j\omega + jn\Omega)|^2 < 0.02277 \quad n \neq 0 \quad (3.28)$$

necessitating a suppression of the sidebands by not less than 16 dB which is not obtainable e.g. by filters of 3rd order

for a cut-off frequency/sampling frequency ratio

$$\frac{\omega_C}{\Omega} = \frac{3.4}{8} \quad , \quad (3.29)$$

as is the case for PCM recovery filters.

On the other hand, for conversion circuits of sufficiently high order, it is possible to guarantee small attenuation in the passband and it is to be expected that the suppression of the sidebands (attenuation in the stopband) is higher than the corresponding values obtained by constant doubly-terminated filters. This is due to the conversion mechanism; indeed, forward attenuation is created not only due to reflection at the input, as in constant filters, but also due to energy appearing at the output at frequencies other than the one under consideration. Efficient transmission in the required passband will have as a result that the remaining energy in the other sidebands is very small.

3. Expressions (3.14) for the transfer function $T(p)$ and (2.96) for the conversion coefficient $S_o(p)$ show a formal similarity. The two expressions are rewritten here for convenience:

$$S_o(p) = 2 \sqrt{R_C R_2} \frac{M(p)}{R_C + Z(p)} \quad (2.96)$$

$$T(p) = 2 \sqrt{R_1 R_2} \frac{M(p)}{R_1 + Z_1(p)} \quad (3.14)$$

The network F in a doubly-terminated filter circuit (Fig. 3.1) plays the role of a matching network between the terminating resistances R_1 and R_2 , while in the case of conversion circuits the matching is now to be achieved between R_2 and R_C , which depends itself on element values of N . Thus, direct use of filtering networks already designed for operating between two terminating resistances did not seem to be a promising approach for designing conversion circuits because of the dependence of R_C on both the input capacitances C and the sampling frequency. Even to use them as a first order approximation implies a number of difficulties and restrictions, however, an algorithm is described in Appendix 1 which may be

successful to modify the parameters of a doubly-terminated filter to yield good conversion circuits which are not necessarily optimum.

The difference between $Z_1(p)$ and $\tilde{Z}(p)$ in the passband is of minor importance if the sampling frequency is sufficiently high (e.g. 10 times) compared to the cut off frequency of the filter [6]. Even for the case when the sampling frequency is slightly higher than double the cut off frequency, the difference between $Z_1(p)$ and $\tilde{Z}(p)$ for a certain filter network is practically negligible for filters of sufficiently high orders. This fact was successfully used to design optimum conversion filters for direct resonant transfer circuits [7] because of the absence of the term R_C in the expressions for the conversion coefficients $S_n(p)$.

4. The minimum sensitivity property of doubly-terminated filter circuits can be generally applied to any circuit designed on the maximum power transfer principle. In particular, this can be directly applied for our conversion circuits to guarantee meaningful application of these in practical situations where the low sensitivity property is vital. However, for the conversion circuits, the minimum attenuation in the passband cannot reach the value zero for circuits built with a finite number of elements, but the low sensitivity behavior can be obtained by choosing a small value for the highest attenuation in the passband, A_d . The design examples given in Chapter 4 have the low sensitivity property as is confirmed by their sensitivity analysis with respect to changes in circuit elements.

3.2 Effective (Insertion) loss design:

We present now a general design method in which networks with a finite number of elements are optimally dimensioned to produce conversion circuits with:

- a) Maximum amount of suppression (attenuation) of the unwanted sidebands.
- b) Minimum sensitivity of their attenuation behavior with respect to variations in the element values.

The formulation is schematically shown in Fig. 3.5 with the attenuation A as defined in equation (3.23) or (3.24). The low sensitivity property is obtained when the maximum allowable attenuation in the passband (A_d) is kept sufficiently small. The minimum attenuation in the

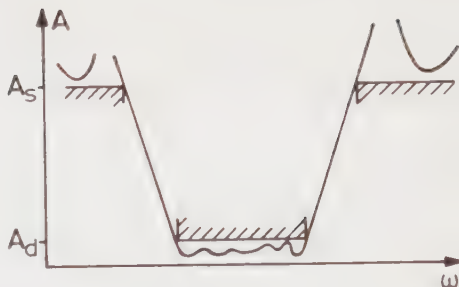


Fig. 3.5

stopband(s) is A_s . This value is assumed to be the highest attainable value when we speak about the optimum design. Clearly, these conditions have to be presented in a suitable mathematical form to allow the application of numerical optimization techniques. This can follow by expressing the conversion coefficient $S_o(p)$ - or equivalently $S_o(\lambda)$ - in terms of a finite number of parameters. Care must be taken to ensure the realizability of the network N as a lossless two-port when the parameters are optimally chosen.

As we are interested in an optimum solution for practical use, we shall limit our study to the case in which the lossless two-port network N , when terminated by R_2 at port 2, exhibits an input impedance Z at port 1 which is normal [1,8,9]. The impedance Z is normal if

a - all the poles of Z are simple.

If we require also that Z is minimum reactive, we add the condition

b - Z has no poles on the $j\omega$ axis.

Using the normalized quantities defined in Section 2.4.1, we can write the conversion coefficient in the form (with $R_o = R_2$):

$$S_n = 2 \sqrt{r_c} \frac{m(\lambda + jn\pi)}{(\tilde{z} + r_c)} . \quad (3.30)$$

We write also for z and $\tilde{z}$

$$z = \sum_{i=1}^k \frac{b_i}{\lambda - \lambda_i} \quad (3.31)$$

$$\tilde{z} = \sum_{i=1}^k b_i \frac{1-\phi\phi_i}{\phi-\phi_i} \quad (3.32)$$

and we have for r_C

$$r_C = \sum_{i=1}^k b_i \quad (3.33)$$

The current transfer function $m(\lambda)$ of the network is related to the input impedance by the expression

$$m(\lambda)m(-\lambda) = |z(\lambda) + z(-\lambda)|/2 \quad (3.34)$$

$$= \sum_{i=1}^k b_i \lambda_i / (\lambda^2 - \lambda_i^2) \quad (3.35)$$

The minimum reactance property of $z(\lambda)$ means that all the poles λ_i of $z(\lambda)$ have a negative real part. The residues b_i corresponding to complex conjugate poles are themselves complex conjugate. For reciprocal two-port networks, the current transfer ratio $m(\lambda)$ can be written in the form

$$m(\lambda) = \frac{f(\lambda)}{d(\lambda)} \quad (3.36)$$

with $f(\lambda)$ either an even or an odd polynomial in λ and $d(\lambda)$ a Hurwitz polynomial. Comparing now equation (3.35) with (3.36) we can write

$$\pm f^2/dd_* = \sum_{i=1}^k b_i \lambda_i / (\lambda^2 - \lambda_i^2) \quad (3.37)$$

$$\text{with } d_*(\lambda) = d(-\lambda) \quad (3.38)$$

We now assume more precisely that $z(\lambda)$ is strictly normal [1], i.e. that

c - the Hurwitz polynomial $d(\lambda)$ in (3.36), for $f(\lambda)$ even or odd, has only simple zeros.

Note that if $f(\lambda)$ has all its zeros on the $j\omega$ -axis, the requirement of strict normality is automatically fulfilled if $z(\lambda)$ is normal. In the following discussion we will also call N to be strictly normal if the input impedance $Z(p)$ - or $z(\lambda)$ - is strictly normal. According to our assumptions, all the λ_i are

distinct and consequently also all the λ_i^2 . Thus no terms can cancel in the summation of equation (3.35), and the Hurwitz polynomial is equal to

$$d = \prod_{i=1}^k (\lambda - \lambda_i) \quad (3.39)$$

If $f(\lambda)$ and $d(\lambda)$ are known, a partial fraction expansion of (3.35) determines the value of the residues b_i

$$b_i = \frac{1}{\lambda_i} f^2(\lambda_i) / \lambda_i \prod_{\substack{q=1 \\ k \neq i}}^k (\lambda_i^2 - \lambda_q^2) \quad (3.40)$$

Equation (3.40) is the key to a parametric representation of the quantities z , $\tilde{z}$, r_C and the conversion coefficient S_n . In light of the foregoing discussion it is possible now to state the realizability conditions for the network N in form of the following theorem

Theorem

Necessary and sufficient conditions for a function $S_n = S_n(\lambda)$ to be the conversion function of order n of a circuit of the type shown in Fig. 2.2, where N is strictly normal, are the following:

1. The variable λ is the normalized complex frequency defined by equation (2.69)
2. The analytic expression of S_n can be put into the form (2.96) or equivalently (3.30) where m is of the form (3.36) and $\tilde{z}$ of the form (3.32) and where r_C is a positive constant given by equation (3.33)
3. In equation (3.36), $d(\lambda)$ is a polynomial of the form (3.39), where the k parameters λ_i are distinct and such that $\text{Re}(\lambda_i) < 0$; complex λ_i occur in conjugate pairs. If λ_i is written in the form

$$\lambda_i = \alpha_i + j\beta_i \quad (3.41)$$

then, with δ_i a real nonzero parameter

$$\alpha_i = -\delta_i^2 \quad (3.42)$$

4. In equation (3.36), $f(\lambda)$ is an arbitrary even or odd real polynomial of degree less than k .
5. The parameters b_i are given by equation (3.40)
6. The variable ψ and the parameters ψ_i are related to λ and the λ_i by equation (2.72).

Proof

The necessity of the conditions is obvious from the above discussion. To prove sufficiency, we assume the choice of the parameters λ_i and the polynomial f in accordance with items 3 and 4 of the theorem. It is then always possible to find a realizable reactance two-port N , which when terminated by a resistance at port 2, shows a current transfer ratio I_2/I_1 equal to f/d and an input impedance z ; with z minimum reactive (and strictly normal because all λ_i are distinct) [10,11]. The input impedance is determined by means of the summation (3.31) where the residues b_i are determined from equation (3.40).

It is thus already established that the input impedance of N is a Brune function and is capacitive at high frequency. This implies that the parameter r_C of equation (3.33) is real and positive. With the network N now having the properties mentioned, and with the parameters r_C, z, m now defined, the pulse impedance $\bar{z}$ can be determined when property 6 holds. Thus the conversion coefficient of order n of the circuit of Fig. (2.2) is identical to the function given by equation (3.28). Q.E.D.

The selection of the polynomial f completes the choice of the arbitrary parameters. The zeros of f are responsible for the zeros of transmission and are chosen to be located either at the origin or at nonzero frequencies on the $j\omega$ -axis. Usually, the zeros of the f polynomial are distributed in the stopbands (in case of recovery of the baseband signal, the network is designed to exhibit low-pass behavior and there is only one stopband). The general form of the f polynomial is then

$$f(\lambda) = k_f \lambda^L \prod_{i=1}^q (\lambda^2 + \Omega_i^2) \quad (3.43)$$

$$\text{with } L + 2q < k \quad (3.44)$$

The constant multiplier k_f amounts to an impedance scaling and has no influence upon the expression of S_n , so that it can always be set equal to unity without any loss of generality. The number of design parameters is now

$$N_d = k + q . \quad (3.45)$$

These are the q finite transmission zeros Ω_i and the k distinct zeros λ_i . For a real λ_i there corresponds a single real quantity while for every complex conjugate pair λ_i, λ_{i+1} there correspond the two parameters α_i and β_i where

$$\lambda_i = \alpha_i + j\beta_i \quad (3.46)$$

$$\lambda_{i+1} = \alpha_i - j\beta_i .$$

As there is no simple way to start the design with an appropriate first approximation obtainable from the known filter synthesis method (as was pointed out in section 3.1), a general design algorithm was developed to compute the optimum choice of the N_d design parameters.

3.3 Optimum conversion circuits:

The conversion circuit is required to exhibit a small attenuation in the passband and the maximum possible amount of suppression of the unwanted side bands (maximum attenuation in the stopbands). Clearly we have to choose an appropriate iteration technique which is able to change the design parameters to yield circuits with the best obtainable behavior. We use a method suggested by Waren and Lasdon [12,13,14] which is reasonably efficient in designing circuits required to meet practical specifications.

It was found in the last section that the conversion coefficient S_n can be determined in terms of the design parameters λ_i and Ω_i . We consider that these parameters constitute the vector $\underline{x}$ comprising the k distinct λ_i 's and the q transmission zeros Ω_i

$$\underline{x} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_k \\ \Omega_1 \\ \vdots \\ \Omega_q \end{pmatrix} \quad (3.47)$$

We designate

$$A_i(\underline{x}) = A(\underline{x}, v_i) \quad (3.48)$$

as the attenuation calculated for the given set of design parameters $\underline{x}$ at the normalized frequencies v_i . It is required now that

$$A_i(\underline{x}) \leq A_d \quad \text{for all } v_i \in \text{PB} \quad (3.49)$$

$$A_i(\underline{x}) \geq A_s \quad \text{for all } v_i \in \text{SB} \quad (3.50)$$

A_d and A_s are respectively the maximum allowable attenuation in the passband and the minimum required attenuation in the stopband(s) (see Fig. 3.5). A first choice of the design parameters λ_i and Ω_i (or, equivalently of the real parameters $\alpha_i, \beta_i, \Omega_i$) is not restricted and is chosen intuitively. The Ω_i 's have to be appropriately distributed in the stopband(s) while the λ_i 's have to lie to the left of the passband as shown in Fig. 3.6. This first guess for $\underline{x}$ leads to a certain attenuation behavior with

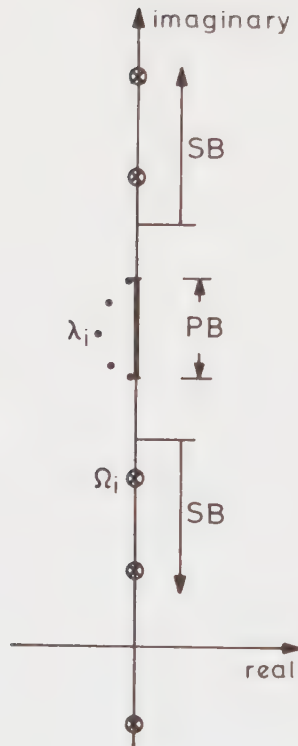


Fig. 3.6

Distribution of the design parameters in the p-plane

distinct pass- and stopbands, but not necessarily satisfying the conditions (3.49) and (3.50) at all frequencies.

There may be many vectors $\underline{x}$ satisfying inequalities (3.49) and (3.50), i.e., many satisfactory designs, or there may be none at all for a certain order k of the filter. In either case, a best design can be meaningfully defined as:

- a) One which maximizes the minimum amount by which the specifications are exceeded.
- b) One which fails to meet the specifications but minimizes the maximum amount by which this is the case.

The solution to the following nonlinear programming problem represents such an optimum design:

$$\begin{aligned} &\text{Determine } (\underline{x}, \xi) \text{ to} \\ &\text{minimize } \xi \end{aligned} \tag{3.51}$$

subject to

$$\begin{aligned} A_i(\underline{x}) - \xi &\leq A_d & v_i &\in \text{PB} \\ A_i(\underline{x}) + \xi &\geq A_s & v_i &\in \text{SB} \end{aligned} \tag{3.52}$$

where ξ is an additional independent variable.

For any vector $(\underline{x}, \xi)$ satisfying (3.52) with $\xi < 0$, the specifications are exceeded at least by $|\xi|$. If $\xi < 0$ and at least one of the constraints (3.50) is binding (i.e., satisfied as an equality), then $|\xi|$ is the minimum amount by which the specifications are exceeded. If $\xi > 0$ and at least one of the constraints is binding, then ξ is the maximum amount by which the specifications fail to be satisfied. Fig. 3.7a shows the case $\xi < 0$ and Fig. 3.7b the case $\xi > 0$ and one constraint is binding.

At a minimum of the above problem, at least one of the constraints (3.52) must be binding, otherwise ξ could still be reduced without violating any of the constraints. Thus, if $\min \xi < 0$,

the solution defines an optimal design which maximizes the minimum amount by which the specifications are exceeded. Similarly, if $\min \xi > 0$, the specifications cannot be met at all points

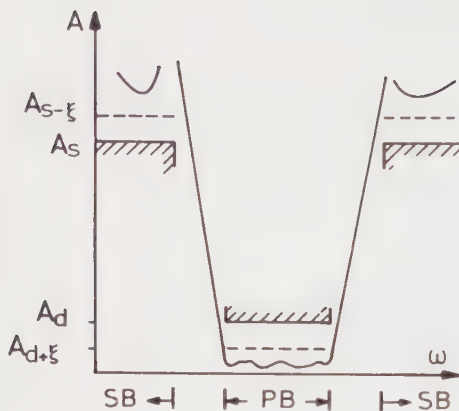


Fig. 3.7a $\xi < 0$ and no constraint binding

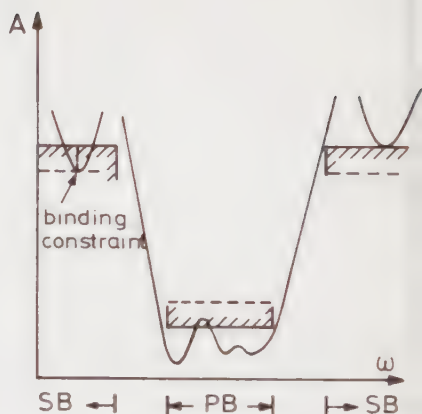


Fig. 3.7b $\xi > 0$ and one constraint binding

and the resulting optimal design minimizes the maximum amount by which the specifications are violated.

More flexibility can be obtained by replacing the constraints (3.52) by

$$\begin{aligned} A_i(\underline{x}) - W_{pi}\xi &\leq A_d & v_i &\in PB \\ A_i(\underline{x}) + W_{si}\xi &\geq A_s & v_i &\in SB \end{aligned} \quad (3.53)$$

where the W_{si} and W_{pi} are non-negative weighting factors. Even more flexibility is obtained if A_s and A_d are replaced by A_{si} and A_{di} , i.e. if the attenuation specifications are allowed to be dependent on the specific value v_i under consideration. During the minimization process, the requirements in the pass-band are heavily weighted at first. Once they are satisfied, i.e., if $\xi < 0$ and

$$A_i(\underline{x}) < A_d \quad v_i \in PB \quad (3.54)$$

the weighting factors W_{pi} are set equal to zero and the minimization is resumed to reach the maximum possible value of attenua-

tion in the stopband.

In carrying out the minimization process, it is sufficient to consider only the behavior at a limited number of frequency points ν_i . The conversion coefficient S_o and the attenuation A are continuous in the frequency ν and the satisfaction of the specifications at a sufficient number of discrete points ν_i implies satisfaction everywhere.

3.4 Solution Technique:

The chief difficulty in solving the minimization problem (3.51) is due to the presence of the constraints (3.52) or (3.53) if weighting factors are used. For a filter of resonable size, the conversion coefficient S_o is highly nonlinear. Thus, a solution technique which works well on problems with markedly nonlinear constraints is needed. It is also desirable that the technique decreases ξ monotonically, while maintaining all constraints satisfied, since this gives an improved filter at every iteration. The technique developed by Fiacco and Mc Cormick [15] has these properties and was found powerfull in solving our problem. This method and an associated minimization technique developed by Powell [16] are described here briefly. For details and usefull hints helping in the construction of a good computer program we refer to [17-24].

3.4.1 The Sequential Unconstrained Minimization Technique:

This technique (abreviated as SUMIT) by Fiacco and Mc Cormick solves an inequality constrained minimization problem by converting it to a sequence of unconstrained minimization problems. The constrained problem (3.51)-(3.52) may be formulated as follows:

minimize ξ

subject to

$$g_i(\bar{x}) \geq 0$$

where

$$\begin{aligned} g_i(\bar{x}) &= A_d + W_{pi}\xi - A_i(\underline{x}) & \nu_i \in PB \\ g_i(\bar{x}) &= -A_s + W_{si}\xi + A_i(\underline{x}) & \nu_i \in SB \end{aligned} \quad (3.55)$$

and the new vector $\bar{x}$ comprises the N_d design parameters and the variable ξ .

We consider now the function

$$F(\bar{x}, \gamma) = \xi + \gamma \sum_{i=1}^{N_p} 1/g_i(\bar{x}) \quad (3.56)$$

where γ is a positive constant and the quantities $g_i(\bar{x})$ are those defined at the N_p test points. The feasible region R is defined as the region containing all combinations $\bar{x}$ satisfying (3.53)

$$R = \{\bar{x} \mid g_i(\bar{x}) \geq 0\} \quad i=1, \dots, NP \quad (3.57)$$

its interior R^0 is defined as

$$R^0 = \{\bar{x} \mid g_i(\bar{x}) > 0\} \quad (3.58)$$

R^0 is nonempty since for any $\bar{x}, \xi$ may be chosen large enough to make the quantities (3.55) positive. For a first choice γ_1 of γ , and starting searching from an arbitrary point $\bar{x}_0$ lying in R^0 , we would expect that a minimum of $F(\bar{x}, \gamma_1)$ exists in R^0 since on the boundary of this set, i.e., for some $g_i(\bar{x})=0$, the function $F(\bar{x}, \gamma)$ grows to infinity. Thus the trajectory of steepest descent from the arbitrary starting point $\bar{x}_0$, cannot cross the boundary of R . The minimizing point depends on the choice of γ_1 and is denoted by $\bar{x}(\gamma_1)$. By the above arguments, $\bar{x}(\gamma_1) \in R^0$.

Now, the minimizing process is repeated for a sequence of values $\gamma_1 > \gamma_2 > \gamma_3 \dots \gamma_k \dots > 0$. The previous minimizing point is used as starting point for the next minimization step. Each minimizing point $\bar{x}(\gamma_k)$ will be in R^0 . Further, by reducing γ the influence of the term

$$\gamma \sum_{i=1}^{NP} 1/g_i(\bar{x}) \quad , \quad (3.59)$$

which penalizes closeness to the constraint boundaries, is reduced and in minimizing $F(\bar{x}, \gamma)$ more effort is concentrated upon reducing ξ . One would expect that as γ approaches zero, the minimizing point $\bar{x}(\gamma)$ approaches the solution to the original

problem (3.51) - (3.52).

3.4.2 Unconstrained Minimization:

In order that the sequential unconstrained minimization technique discussed above be effective, an efficient minimization method is required. Recommended methods come from the class of quadratically convergent procedures. The method described by Powell [16] is chosen for our case because it does not require calculating the derivatives of the conversion coefficient with respect to the design parameters. The method depends only on the evaluation of the function values and shows good convergence behavior [16-20].

3.5 The Optimization Program:

A program was written which is based on the aforementioned representation and many other details and modifications; it is characterized by the following:

1. The input data to the program consists of the filter order, number of transmission zeros at the origin (when designed as a bandpass) and the specifications for the attenuation in both the pass- and stopband A_d and A_s .
2. For convenience the program uses the magnitude of the conversion coefficient S_o rather than the attenuation; and the attenuation specifications are translated into specifications on the magnitude of the conversion coefficient. Thus, with

$$S_d = e^{-A_d}, \quad A_d \text{ in Nepers} \quad (3.60)$$

$$\text{and} \quad S_s = e^{-A_s}, \quad A_s \text{ in Nepers} \quad (3.61)$$

used as the specifications rather than A_d and A_s , we can write for the optimization problem:

minimize ξ

subject to

$$g_i(\bar{x}) \geq 0 \quad (3.62)$$

$$\text{where } g_i(\bar{x}) = S_d + W_{Pi}\xi + |S_i(\bar{x})| \quad v_i \in PB \quad (3.63)$$

$$g_i(\bar{x}) = - |S_i(\bar{x})| + W_{Si}\xi + S_s \quad v_i \in SB$$

$$\text{with } |S_i(\bar{x})| = |S_o(v_i)| \quad (3.64)$$

being the magnitude of the conversion coefficient S_o at the normalized frequency v_i . Since the magnitude of the conversion coefficient S_o cannot exceed the value 1 anywhere, the constraints $g_i(\bar{x}) \geq 0$ are certainly all satisfied if we adopt in (3.63) the value 1 as first choice for all quantities $W_{Pi}\xi$ and $W_{Si}\xi$.

3. The starting values for the design parameters (α_i, β_i and Ω_i) are fed to the program together with a first estimation for the parameter γ (usually 1). As a termination criterion for the iterations, the progress in decreasing ξ is used.

4. If the original specifications can be satisfied, the weighting factors W_{Pi} in equation (3.63) are all set equal to zero and the optimization is continued to get filters with the maximum available attenuation in the stopband under the condition that the attenuation in the passband does not exceed its specified value A_d . It is to be expected then that the attenuation reaches the value A_d at distinct frequency points in the passband.

5. The parameter γ is reduced monotonically by a constant reduction factor,

$$\frac{\gamma_l}{\gamma_{l+1}} = k_\gamma, \quad (3.65)$$

and the point of the minimum, $\bar{x}(\gamma_l)$ is used as a starting point for the next minimization step. Within very wide tolerances, the reduction factor k_γ is not very critical to the convergence of the algorithm. Clearly, k_γ must, at any rate, be greater than one, and for every minimization step a sufficient number of iterations (and consequently sufficient calculating time) is necessary. If k_γ is chosen very great, a smaller number of minimization steps is required to obtain the optimum solution, but for every minimization step, a large number of iterations is required and vice versa. However, a good choice is always possible and it depends upon the nature of the specific case and the number of variables of the function to be minimized. We have found that a choice

$k_Y = 100$ has given the best convergence speed when a 5th order lowpass filter (7 design parameters) was optimized. Ten minimization steps were then necessary to get the optimum solution. In the case of a 12th order bandpass filter, with 16 design parameters, a value of 10000 for k_Y was found to be good and a total number of 6 minimization steps was necessary to reach an optimum. 6. As an output, the program delivers the optimum values for the k parameters λ_i and the q transmission zeros Ω_i . The highest value of the minimum attenuation obtainable in the stopband is also available.

3.6 The Calculation of the Circuit Elements:

The output data obtained from the optimization program, namely the optimum values of λ_i and Ω_i ; have to be suitably prepared to enable the use of the available synthesis program ABSPA [2], which calculates the element values of doubly terminated constant circuits - as that of Fig. 3.1 - when the polynomials $f(p)$, $g(p)$ and $h(p)$ are known. A program was written which operates as follows:

1. The current transfer ratio m and the input impedance z_1 are determined by the help of equations (3.31, 3.36, 3.40, and 3.43).
2. An auxiliary arrangement is considered by connecting a resistance r_1 to the network N whose input impedance and current transfer ratio are z_1 and m , respectively. The resulting arrangement is of the same type as the doubly-terminated arrangement of Fig. 3.1.
3. The transfer function $T(p)$ and the reflection factor $\rho_1(p)$ of the auxiliary arrangement are calculated by the help of equations (3.14) and (3.6).
4. The polynomials $h(p)$, $f(p)$ and $g(p)$ are then known (compare equations (3.3) and (3.7)) and can be presented to the synthesis program ABSPA to determine the element values of the network N .

The correct evaluation of the reflection factor $\rho_1(p)$ and the polynomial $h(p)$ is very important to guarantee that the element values, and consequently the value of the input capacitance C , are the correct values corresponding to the given optimum parameters λ_i and Ω_i .

It must be stated clearly that this method was chosen to facilitate the use of the available synthesis program ABSPA. In principle, it is possible to synthesize the network N from its current transfer function M as is stated in [11] if the corresponding synthesis programs were available.

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Chapter 4 : Design Examples

The two cases presented in this chapter are more than mere numerical examples which prove the solution method and the program to function well. They represent a complete design complemented by experimental testing and sensitivity analysis for two important problems. The first is a recovery filter for PCM terminating equipment and represents an alternative solution to those available [1-7]. The second is a novel circuit for PCM/FDM transmultiplexing. The background of the two problems was discussed in Chapter 1 and does not have to be repeated here.

In both cases, the conversion circuit is composed of the lossless network N with the termination R_2 . The driving stage is the same in both cases and was represented in the analysis as an ideal voltage source and an ideal switch. We begin at first by discussing the practical aspects of the nonideal driving stage and the effect of the finite pulse width of the current pulses passing through the switch. We follow then by presenting each case alone considering the individual peculiarities of the attenuation curves, the sensitivity analysis, structure and element values as well as practical results. A comparison between the behaviour of the conversion circuits treated in the design examples and corresponding doubly terminated filters sheds more light on the discussion of section 3.1.2.

4.1 The driving stage:

The driving stage is built with the use of a line driver - an amplifier with very low output impedance - and a field effect transistor. The combination is an approximate realization of the ideal source-switch combination of Fig. 2.2 and has to exhibit in practice low on-resistance compared with the impedance level of the network N . For this purpose, the line driver (LH0002) and the field effect transistor (E105) were found suitable.

The line driver LH0002 is a current amplifier providing wide-band unity voltage gain and peak currents of up to ± 200 m.A. It has an input impedance of 200 K Ω and an output impedance of

6 ohms.

The field effect transistor E105 has an on-resistance of 3Ω and small rise and fall times (each 20 n s).

The line driver is fed from a digital to analog converter (DAC). The output signal of the D/A converter can correspond to a single channel or can even be a composite signal carrying information from e.g. 30 PCM channels serially so that the line driver could feed e.g. 30 distinct networks through 30 field effect transistors. This is schematically shown in Fig. 4.1. A circuit diagram of the driving stage is given in Fig. 4.2a.

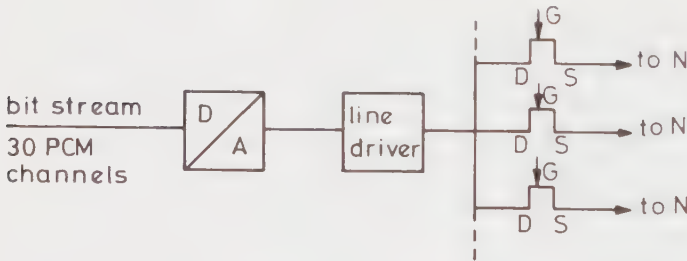


Fig. 4.1 End-stage of 30 PCM channels

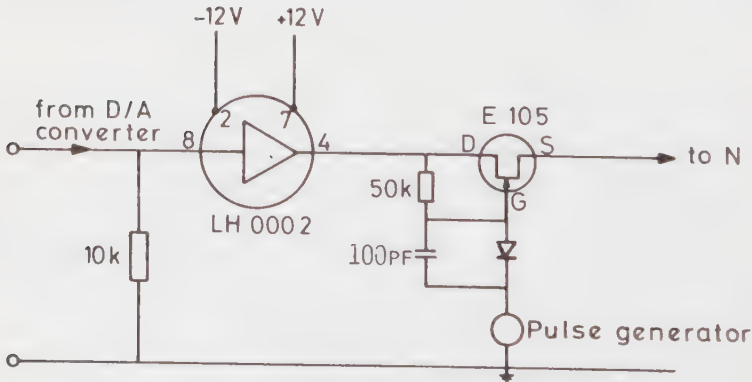


Fig. 4.2a Circuit diagram of the driving stage

When the impedance level of the network N is chosen to be about 1 K Ω , the corresponding input capacitance is about 60 n.F. for circuits fed by 8 KHz samples, and the time constant of the driving stage-input capacitance combination is about 0.5 μ sec. The behavior of the optimum conversion circuits is such that the charge on the input capacitance and consequently the voltage v_b , i.e. the voltage before the arrival of a new current pulse, is nearly zero. This is clear from equations (2.51), (2.86) and (2.94), and is necessary in order that an efficient transfer of energy can take place by fully charging the input capacitance through the switch and fully discharging it into the load resistance R_2 through the network N every period. That this is indeed the case, was confirmed by observing the behavior of the input voltage v_1 (Fig. 2.2), at port 1 of N on the oscilloscope during testing of the optimally designed circuits.

This leads to a very interesting property of the optimum conversion circuits, namely, that their attenuation behavior is minimally effected by the presence of a finite nonzero resistance in the driving stage. It is expected that the charge flowing through the switch during the finite switch closure duration τ is not sufficient to charge the input capacitance completely to the source voltage or to clear the capacitance from the residual voltage on it, v_b . By noting that the residual voltage v_{1b} (and V_{1b}) across the capacitance before the arrival of new current pulse is practically zero for the optimum conversion circuit, we can write for V_{1a} , the value of V_1 , after switch closure and reopening

$$V_{1a} = nE \quad (4.1)$$

where

$$n = 1 - e^{-\tau/RC_i}, \quad (4.2)$$

and R is the output resistance of the line-driver added to the on resistance of the field effect transistor, as is schematically shown in Fig. 4.2 b.

The factor n is frequency-independent and all currents and voltages in the network N have now to be multiplied by n . The conversion function must read now

$$S'_0(p) = n S_0(p) \quad (4.3)$$

and the attenuation A' to be measured is

$$A' = \alpha_o + A = -20 \lg_{10} \frac{2\sqrt{RC} |V_2'|}{|E|} \quad (4.4)$$

where A is the attenuation in the ideal case as given by

$$A = -20 \lg_{10} |S_o(j\omega)| = -20 \lg_{10} \frac{2\sqrt{RC} |M(j\omega)|}{|RC + \tilde{Z}(j\omega)|} \quad (4.5)$$

V_2' and E are the actual measured output voltages at the output of the driving stage and the open circuit output voltage measured at the output of the line-driver when it is not loaded, respectively. The constant factor η contributes a constant basic loss α_o ,

$$\alpha_o = -20 \lg_{10} \eta \quad (4.6)$$

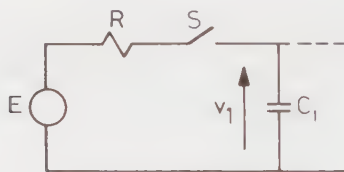


Fig. 4.2b Equivalent circuit of the driving stage

It is very important to manifest that this constant basic loss does not affect in any way the minimum sensitivity property of the practical circuit. The mechanism of energy conversion is not affected by the presence of losses in the driving circuits but the amount of energy is reduced by a factor equal to η^2 . Needless to say, the effect on the shape of the attenuation curve is expected to be minimal.

As the switch closure time τ must be kept sufficiently small to guarantee the validity of the analysis method of chapter 2, a value of 1-2 μs for PCM recovery or PCM/FDM conversion circuits is a good compromise to avoid small values of the factor η and consequently large values of the basic loss α_o . For a time constant RC_1 of 0.5 μs , the corresponding values of η and α_o are:

τ μ s	η	α_0 dB
1.0	0.86466	1.263
1.5	0.95021	0.4435
2.0	0.9816	0.1605

4.2 PCM recovery circuit:

4.2.1 Characteristics of the optimum solution

The network N is designed as a 5th order low-pass filter with 2 transmission zeros at finite frequencies. The sampling frequency is 8 kHz, and the circuit has a passband extending from zero to 3.4 kHz in which the attenuation must not exceed the value $A_d = 0.1$ dB. The minimum attenuation in the stopband (A_s) is required to be as large as possible. The stopband extends from 4.6 to 8 kHz. The following are the optimum values obtained for the normalized circuit parameters:

sampling frequency = π
 cutoff frequency $\nu_c = 1.33517$
 stopband: from $\nu_s = 1.806415$ to π

$$\lambda_1 = -0.3777 + j 00$$

$$\lambda_{2,3} = -0.2316 \pm j 1.0191$$

$$\lambda_{4,5} = -0.0493 \pm j 1.399$$

$$\Omega_1 = 1.8618$$

$$\Omega_2 = 2.5051$$

The optimum value for A_s was found to be 46.9 dB. The theoretical attenuation curve is shown in Fig. 4.3. In the passband, the attenuation reaches the limit A_d at the origin, at the cutoff frequency and at a frequency inside the passband. The behavior of the attenuation curve in the passband is thus reminiscent of a conventional 4th order low-pass Cauer filter. This can be explained by the fact that the number of degrees of freedom is now lower by one. There is indeed only one terminating resistance in the conversion circuit compared to two in the conventional filter circuit case.

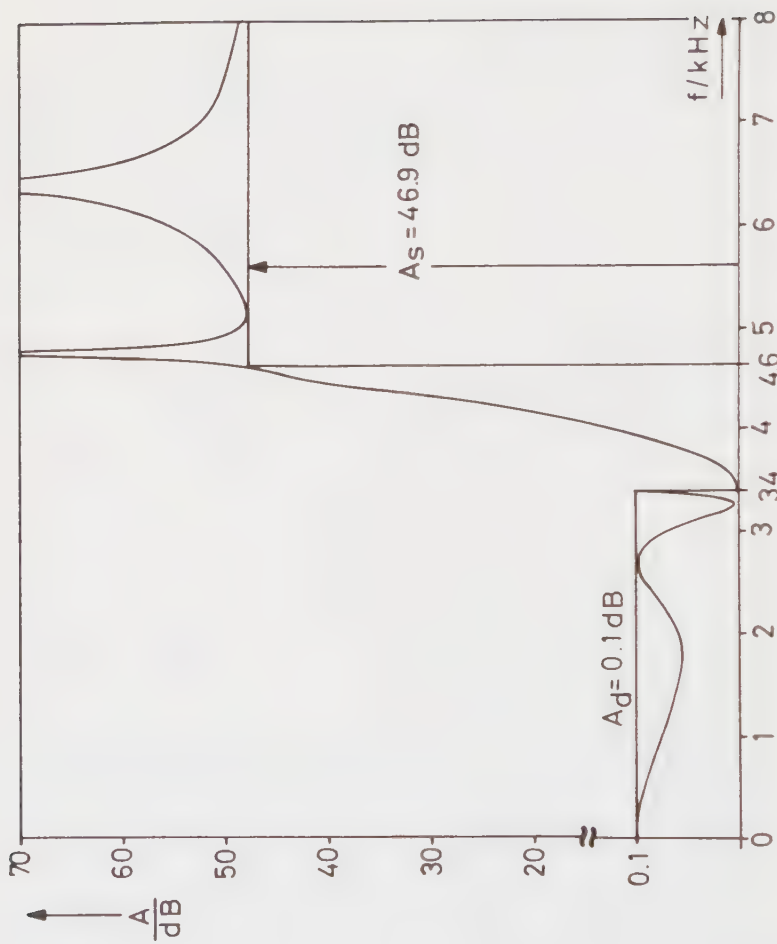
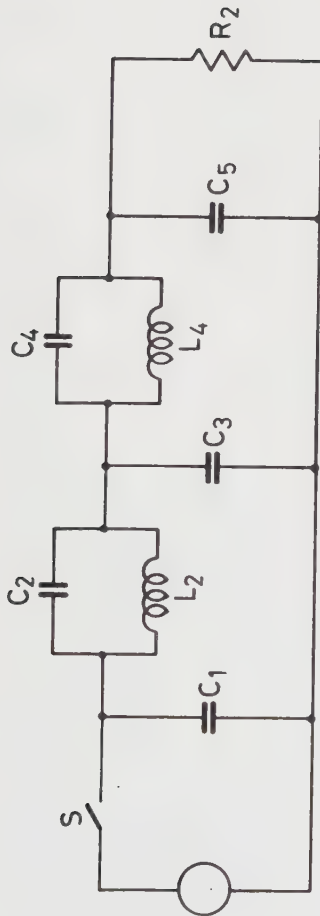


Fig. 4.3 Attenuation curve of the 5th order low-pass recovery filter



Element values

$C_1 = 61.49 \text{ nF}$	$C_4 = 13.25 \text{ nF}$	$L_4 = 46.954 \text{ mH}$
$C_2 = 31.16 \text{ nF}$	$C_5 = 54.82 \text{ nF}$	$R_2 = 1000 \text{ Ohm}$
$C_3 = 78.81 \text{ nF}$	$L_2 = 36.162 \text{ mH}$	

Fig. 4.4 Circuit diagram of the 5th order recovery filter

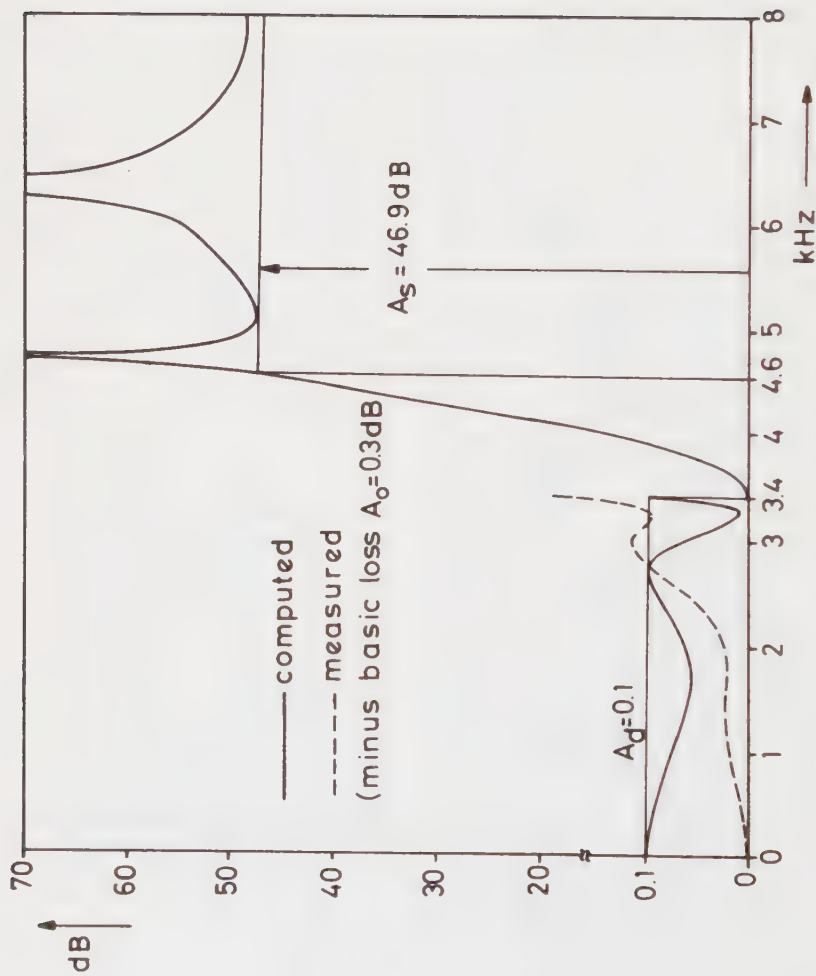


Fig. 4.5a Measured attenuation curve

For experimental verification, the circuit was built according to the structure and element values shown in Fig. 4.4. The circuit was tested by use of the HP3570A network analyzer with pulse width equal to 2 μ s. The measured result is shown in Fig. 4.5a. For ease of comparison between the theoretical (ideal) and practically measured results in the passband, a constant basic loss A_0 of 0.3 dB was subtracted. By noting that about 0.2 dB are unavoidable because of the nonideal behavior of the driving stage and the resulting basic loss α_0 , and that the theoretical curve starts at the origin with 0.1 dB attenuation, it is easy to explain the origin of the 0.3 dB basic loss measured. The slight increase of the attenuation towards the passband edge is mainly due to the losses in the coils; however, the measured attenuation is almost within 0.1 dB above A_0 all over the passband except in the vicinity of the cutoff frequency, where however it is still less than 0.2 dB. The measured attenuation in the stopband agrees well with the calculated one.

Behavior beyond 8 kHz: The region beyond 8 kHz was not considered in the optimization while designing the PCM recovery circuit. As shown in Fig. 4.5b, the minimum attenuation in the region 8-11.4 kHz (43dB) is slightly lower than that obtained for the first side band. Around 12 kHz, the attenuation exhibits a dip to a distinct minimum of 38 dB which is due to the periodic nature of the pulse impedance $\tilde{Z}$. $\tilde{Z}$ has minima at 4 kHz and at $4+n8$ kHz with $n=-2,-1,1,2,\dots$. For the terminal equipment of a PCM channel, this behavior may be of no disadvantage since the human ear is less sensitive for higher frequencies and also because of the low-pass effect of other parts of the equipment (e.g. the earpiece of a telephone set).

However, the region beyond 8 kHz can of course be considered in the optimization process, and it may be required that the minimum attenuation in the stopband (for all frequencies higher than 4.6 kHz) be set as high as possible. The result is then an alternative design with the attenuation behavior shown in Fig. 4.5c. The corresponding (normalized) solution is characterized by:

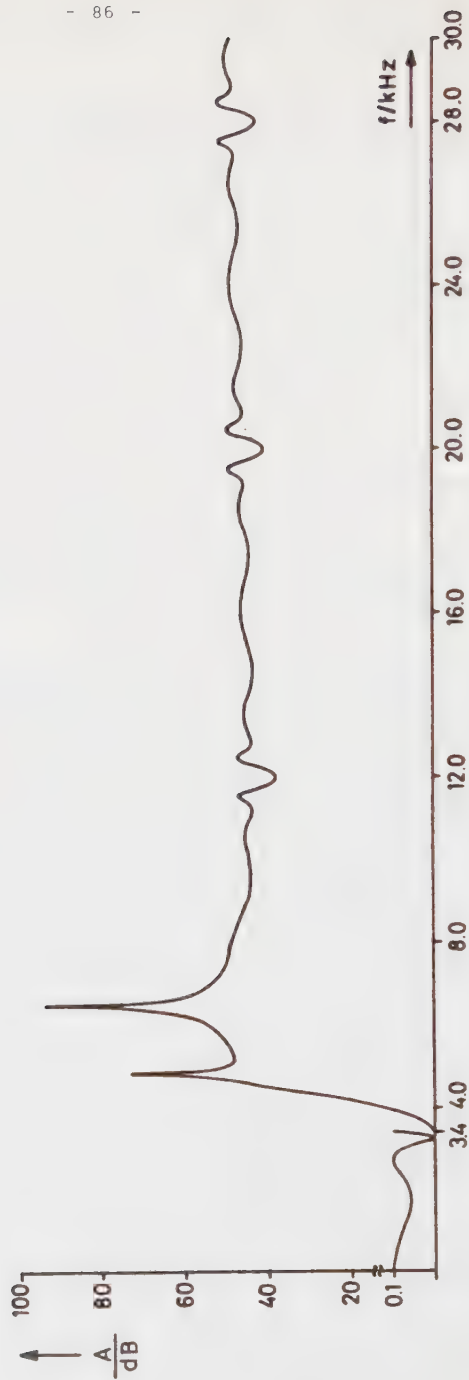


Fig. 4.5b

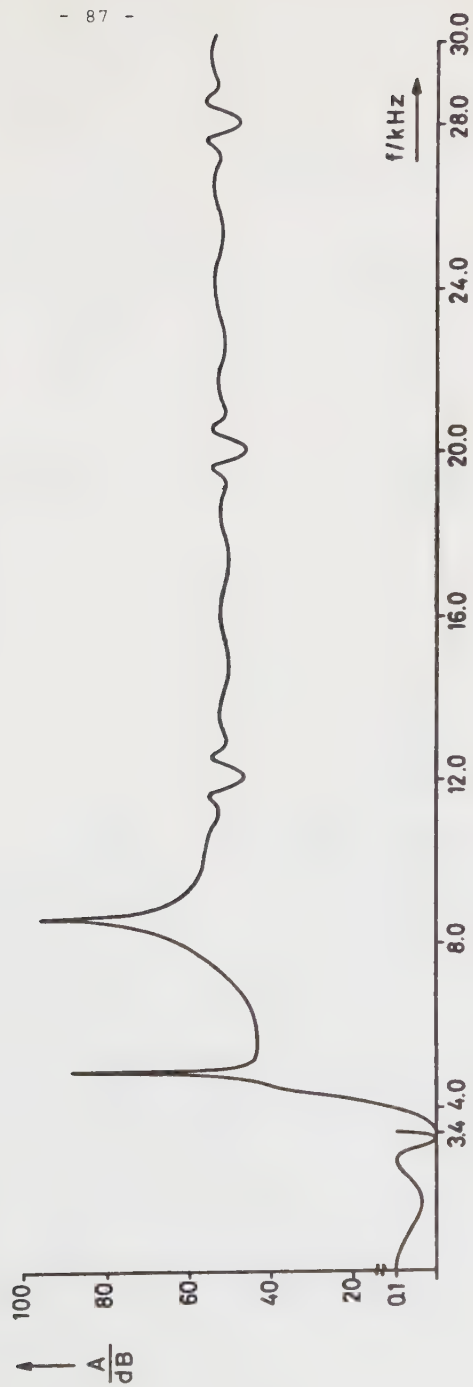


Fig. 4.5c

sampling frequency	= π
cutoff frequency	= 1.33517
stopband edge frequency	= 1.806415
λ_1	= $-0.372039 + j\ 0.0$
$\lambda_{2,3}$	= $-0.245026 \pm j\ 0.985055$
$\lambda_{4,5}$	= $-0.054805 \pm j\ 1.396523$
Ω_1	= 1.87563
Ω_2	= 3.32245
Ad	= 0.1 dB
As	= 43.8 dB

and the element values for the conversion circuit corresponding to that of Fig. 4.4 are:

C_1	= 62.22	n.F.
L_2	= 36.99	m.H.
C_2	= 30.0	n.F.
C_3	= 80.84	n.F.
L_4	= 53.9	m.H.
C_4	= 6.56	n.F.
C_5	= 58.15	n.F.

4.2.2 Sensitivity behavior:

The optimum circuit presented in section 4.2.1 was tested for its sensitivity behavior by calculating the attenuation characteristics of the circuit when one or more elements were changed from their nominal values. Throughout the analysis, the locations of the two attenuation poles were held constant (unvaried). Whenever any one of the capacitances C_2 or C_4 was changed, the suitable coil was also changed so that the resonant frequency and consequently the location of the attenuation pole was held constant. To demonstrate the effect of varying an element, the ideal attenuation curve is compared with the new resulting one. In Fig. 4.6a, the effect of varying the terminating resistance R_2 by $\pm 5\%$ is shown. By inspecting the effect on the attenuation of varying the filter capacitances, it was noticed that the capacitance C_3 affects the attenuation deviation more

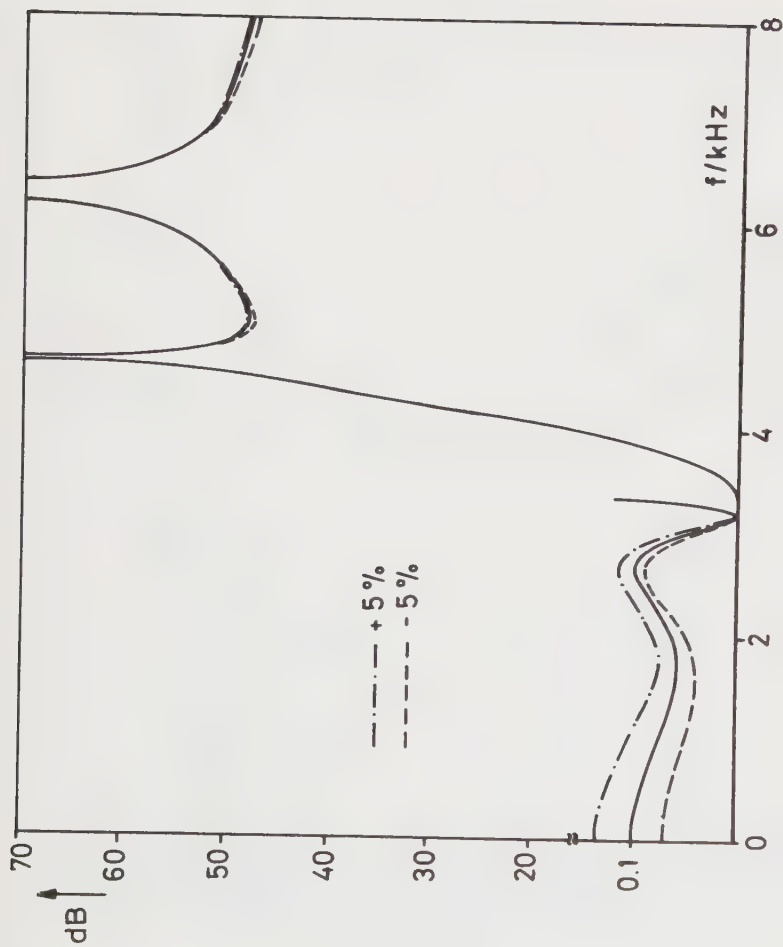


Fig. 4.6a Effect of varying R_2 by $\pm 5\%$

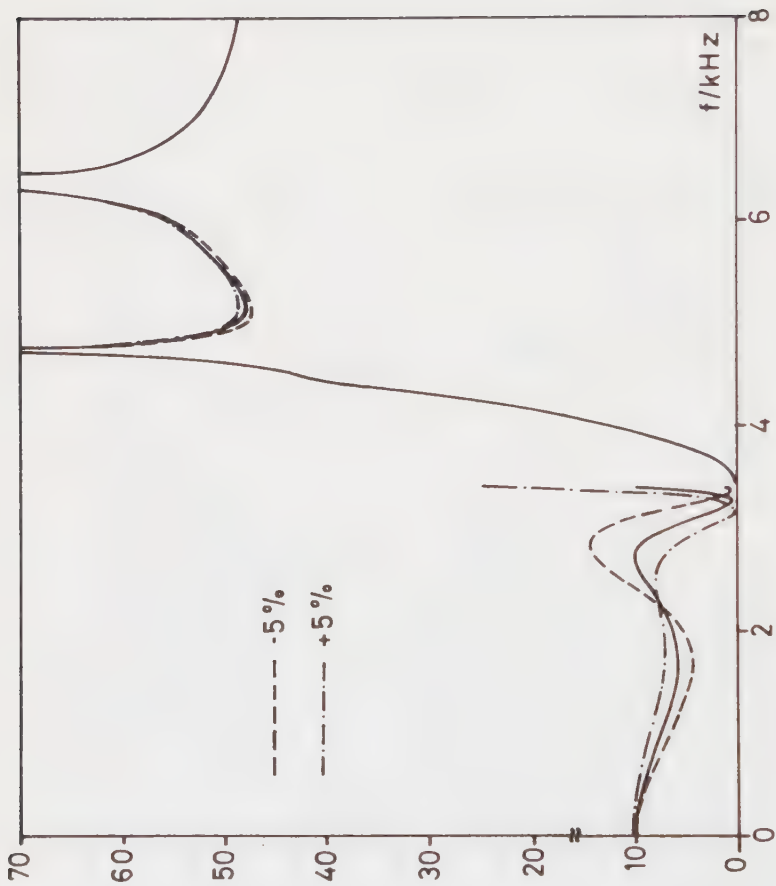


Fig. 4.6b Effect of varying C_3 by $\pm 5\%$

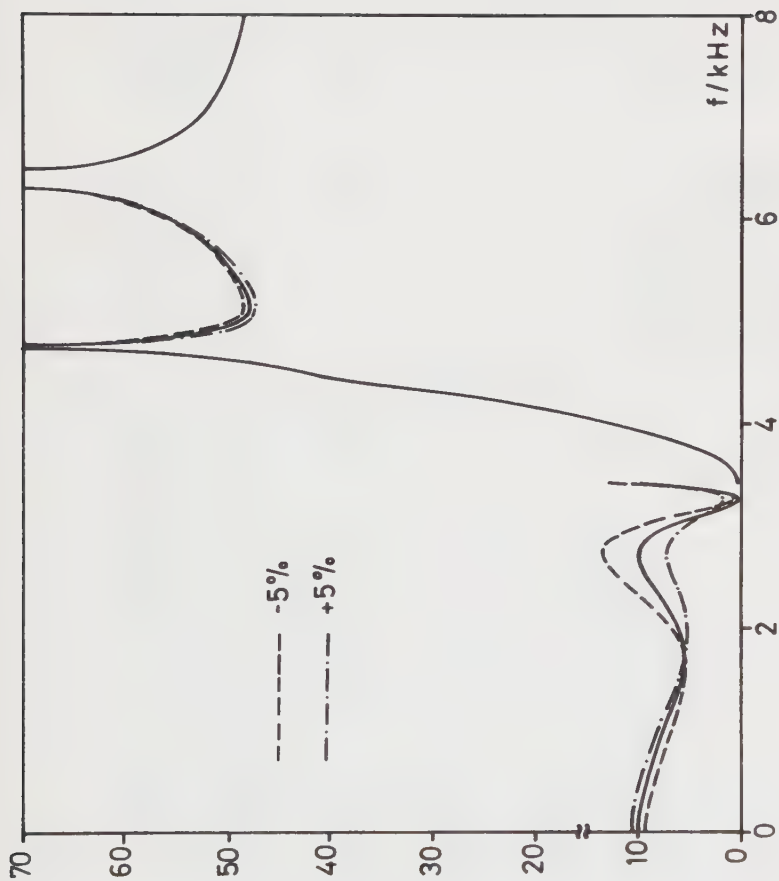


Fig. 4.6c Effect of varying C_2 by $\pm 5\%$

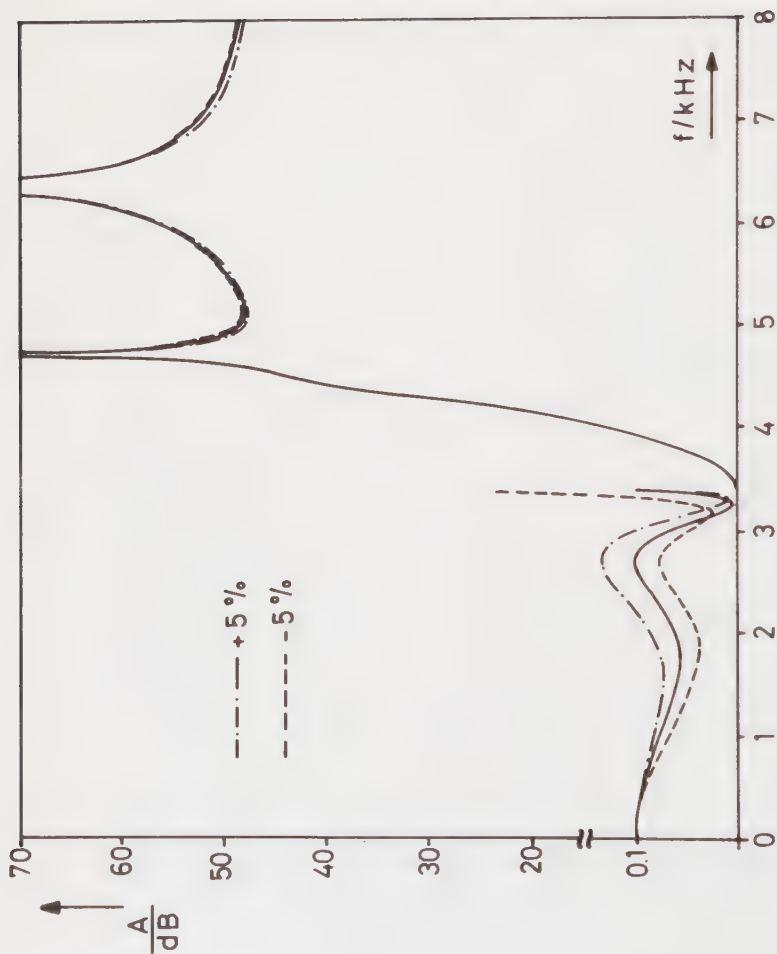


Fig. 4.6d Effect of varying C_u by $\pm 5\%$

than the other ones; however, the results are fairly good and manifest the expectance of minimum sensitivity with respect to element variation, as is shown in Fig. 4.6b. In this Figure, variations of $\pm 5\%$ in C_3 have nearly no effect at 3 points (frequencies) inside the passband, but the effects are noticeable at the cutoff frequency and in parts of the stopband. By similar investigations on the effect of C_2 and C_4 , the results were as shown in Figs. 4.6c and 4.6d; again the minimal sensitivity property is manifested.

The value chosen for testing the sensitivity behavior of the circuit, namely $\pm 5\%$, is clearly an exaggeration to the practical case where elements with e.g. $\pm 1\%$ or $\pm 2\%$ tolerance are used in building such filters. For variations of the order of $\pm 1\%$ or $\pm 2\%$ in the element values, the corresponding deviations of the attenuation were found to be too small to be visible on curves of the size of Fig. 4.6. Changes in the attenuation due to element changes of $\pm 1\%$ are noticeably smaller than one fifth of those obtained for $\pm 5\%$ element changes.

4.2.3 Comparison with doubly terminated networks:

According to the discussion of section 3.1, it is expected that attenuation obtained by a conversion circuit in its stopband will be larger than that of a doubly terminated filter working under similar conditions. To compare the conversion circuit under study, the properties of the 5th order low-pass Cauer filter are given below:

passband	0 - 3.4 kHz
stopband edge	4.6 kHz
Attenuation in passband	≤ 0.1 dB
minimum attenuation in stopband	37 dB .

The attenuation curves of both circuits are shown in Fig. 4.7. A difference of about 10 dB is clearly visible in the stopband.

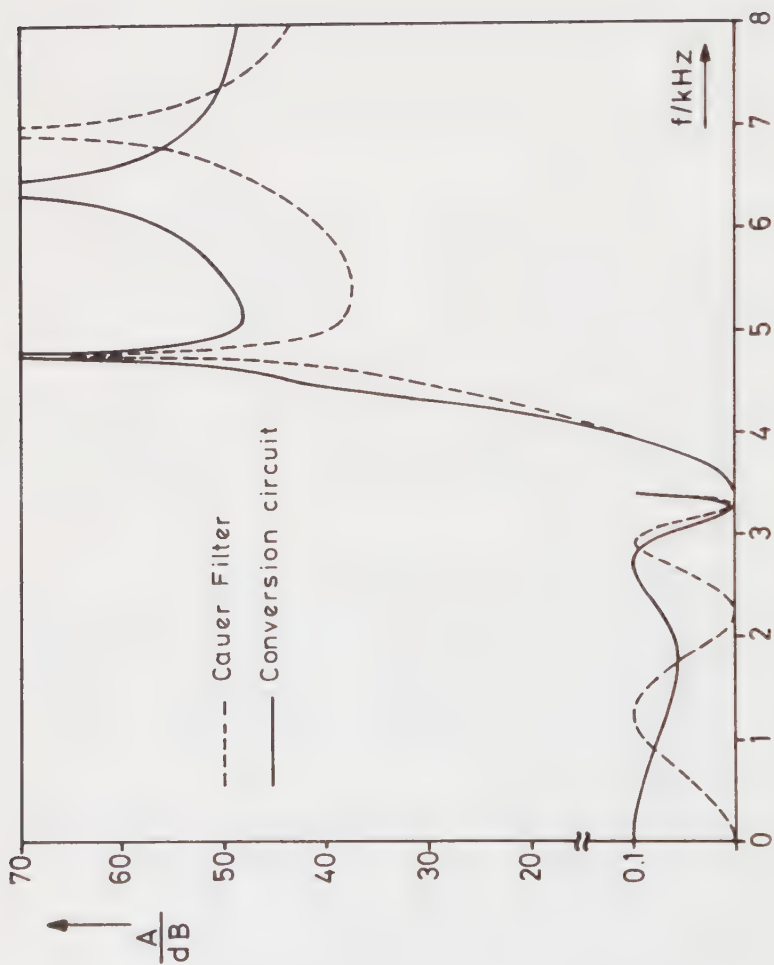


Fig. 4.7 Comparison between doubly-terminated cauer filter and the conversion filter

Note that for the Cauer filter of Fig. 4.7 the attenuation does not decrease below 37 dB also for any frequency beyond 8 kHz, while the conversion filter considered have been optimized only for a stopband frequency range extending up to 8 kHz. Another comparison may thus be made with another doubly terminated filter which is similarly optimized for a stopband range extending from 4.6 to 8 kHz [10]. The minimum stopband attenuation in this range is then found to be 38.6 dB (attained at 4.6 kHz, 5.26 kHz and 8 kHz), while an absolute minimum of 35 dB is reached at 12 kHz.

4.2.4 Comparison with available solutions:

It is difficult to make a direct comparison between the known circuits used for PCM recovery and the results obtained in our work. This is due to the fact that most of the published work in this area is based on other ideas, where the recovery circuit must e. g. compensate for the $(\sin x)/x$ effect of a sample and hold circuit [2,3,4,5] or where the sensitivity problem is treated as one of the bases of the design strategy [4].

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Another difference stems from the fact that the published results treat active circuits designed by cascading several low order (2 or 3) stages [3,4]. Some incomplete results are available for LC filters [6,8].

By noting that the conversion circuit presented here is a fairly insensitive filter of order 5 having a maximum passband attenuation of 0.1 dB and exhibiting a minimum suppression of the unwanted sideband by 46.9 dB, we can make the comparison in the light of the mentioned properties. The results obtained by Schultze [8] are based on a similar circuit arrangement as in our case, i.e. that the circuit is assumed to be fed by narrow current pulses at its input, but the method of analysis differs substantially from ours. The transfer function considered is based on comparing the output voltage with the original signal voltage from which the pulses are obtained, but power relations as in our analysis were not used to judge the effective insertion loss of the filtering circuit. As a design example, a 6th order filter is given in [8], having one attenuation pole at the origin and two in the stopband ($f \geq 4.6$ kHz). The passband extends from 0.3 kHz to 3.4 kHz, and the upper limit of the allowed attenuation is 0.25 dB. The corresponding minimum suppression in the stopband is only 44 dB. Nothing is given in [8] about the power relations or the sensitivity behavior of the circuit.

The number of active circuits used for PCM recovery is quite large; as a representative example we consider the circuit presented by Mikhael and Keys [4]. This circuit is composed of a 3 stage 7th order filter whose transfer function is predistorted to compensate for the $(\sin x)/x$ effect. Special precautions are made in the choice of the design parameters to keep the sensitivity behavior of the circuit acceptable. The theoretical attenuation curve is such that in the passband the attenuation is kept within ± 0.1 dB, and the optimized circuit exhibits about 40 dB suppression in the stopband.

Recently, Nakhla [5] gave a design method based on approximating the filter transfer function to accommodate a correction of the $(\sin x)/x$ factor. As an example, a 7th order filter

circuit containing 4 coils and 5 capacitances is given with the resulting attenuation curve. In the passband a ripple of ± 0.05 dB is imposed, and the resulting suppression is about 38 dB in the stopband. Nothing is known about the sensitivity behavior of the circuit, but we recall here that the ratio between the terminating resistances is $R_1:R_2 = 10:1.93$; meaning that a high basic loss of 7.14 dB is unavoidable.

4.3 PCM/FDM conversion circuit:

4.3.1 Characteristics of the optimum solution:

The network N is designed as a band-pass filter with cut-off frequencies 16.3 and 19.4 kHz, thus corresponding to a channel filter for the middle channel in the 12-24 kHz pregroup of a carrier system. The order of the filter was chosen to be 12 and it has zeros of transmissions at four finite nonzero frequencies. The optimum normalized values of the design parameters were found to be:

$$\begin{aligned}\lambda_{1,2} &= -0.01636636417 \pm j 6.37809869 \\ \lambda_{3,4} &= -0.07958814936 \pm j 6.49892342 \\ \lambda_{5,6} &= -0.138395645 \pm j 6.778116771 \\ \lambda_{7,8} &= -0.15451841525 \pm j 7.176914623 \\ \lambda_{9,10} &= -0.110332409 \pm j 7.49835732 \\ \lambda_{11,12} &= -0.020914571 \pm j 7.66975027 \\ \Omega_1 &= 5.7974249 \\ \Omega_2 &= 6.1281586 \\ \Omega_3 &= 8.144510525 \\ \Omega_4 &= 8.89525929 \\ A_d &= 0.1 \text{ dB} \\ A_{s1} &= 60 \text{ dB} \quad , \text{ lowest attenuation in the lower} \\ &\quad \text{stopband (below 15.7 kHz)} \\ A_{s2} &= 65 \text{ dB} \quad , \text{ lowest attenuation in the upper} \\ &\quad \text{stopband (above 20.6 kHz)}\end{aligned}$$

The corresponding computed attenuation curve is shown in Fig. 4.8. In the passband, the shape of the attenuation curve is reminiscent of that of an equiripple curve of a doubly terminated

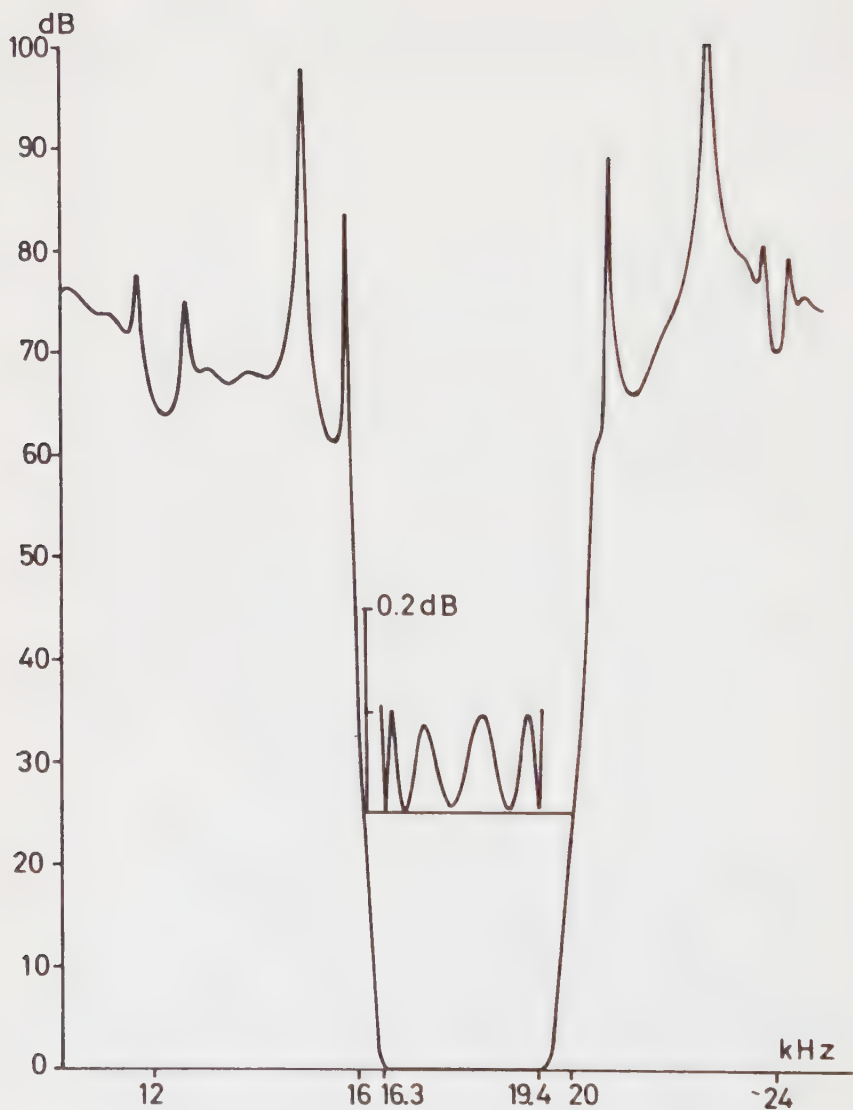


Fig. 4.8 Attenuation curve of the 12th order band-pass conversion filter

filter of order 10 rather than 12. The values which the attenuation takes at the cutoff frequencies and at the four passband maxima are all nearly equal to one another. It can fairly be assumed that they could all be made to approach arbitrarily closely the maximum permitted value of 0.1 dB by carrying out a number of additional iteration steps, assuming that the grid density for the test points and the numerical accuracy are sufficiently high to make further iterations meaningful.

For experimental verification, the structure and element values were calculated according to the method given in Chapter 3 and with the help of the program ABSPA1 [9]. The result was the coil-saving structure shown in Fig. 4.9. The capacitances C_2 , C_4 , C_6 and C_8 together with the respective coils to which they are connected (either in series or in parallel) are responsible for the transmission zeros. On the other hand, the combinations L_1 , C_1 and L_6 , C_{10} resonate at the frequencies

$$\begin{aligned}\omega_{01} &= 1/\sqrt{L_1 C_1} \\ \omega_{02} &= 1/\sqrt{L_6 C_{10}}\end{aligned}\tag{4.7}$$

which are located inside the passband. The element values which have been obtained are as follows

$$\begin{aligned}C_1 &= 61.078977 \text{ n.F.} \\ C_2 &= 4.023031 \text{ n.F.} \\ C_3 &= 2.517034 \text{ n.F.} \\ C_4 &= 3.826881 \text{ n.F.} \\ C_5 &= 14.792674 \text{ n.F.} \\ C_6 &= 10.838331 \text{ n.F.} \\ C_7 &= 3.819505 \text{ n.F.} \\ C_8 &= 9.092814 \text{ n.F.} \\ C_9 &= 21.344504 \text{ n.F.} \\ C_{10} &= 1.571362 \text{ n.F.} \\ L_1 &= 1.265287 \text{ m.H.} \\ L_2 &= 12.271267 \text{ m.H.}\end{aligned}$$

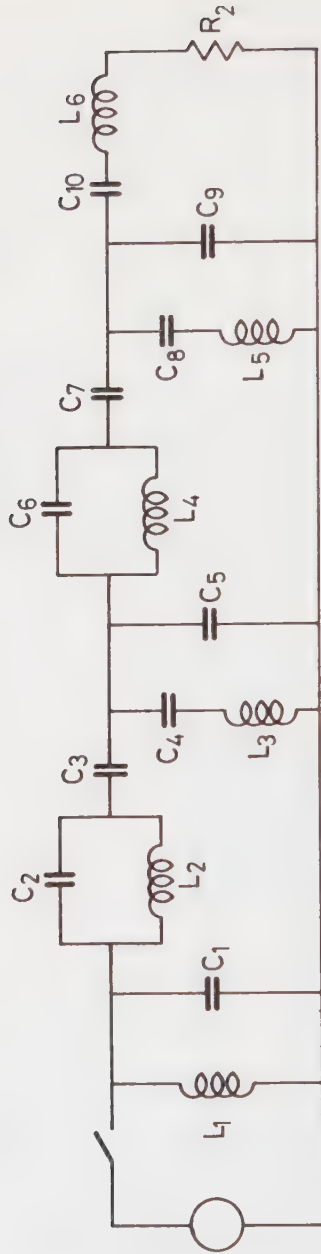


Fig. 4.9 Circuit diagram of the 12th order conversion filter

$$L_3 = 27.180349 \text{ m.H.}$$

$$L_4 = 5.433349 \text{ m.H.}$$

$$L_5 = 12.781782 \text{ m.H.}$$

$$L_6 = 53.094261 \text{ m.H.}$$

$$R_2 = 888.684677 \text{ Ohm}$$

a circuit corresponding to these elements was built with coils whose quality factors, measured at 18 kHz, are

$$Q_1 = 868 \qquad Q_2 = 1030$$

$$Q_3 = 1200 \qquad Q_4 = 909$$

$$Q_5 = 1155 \qquad Q_6 = 909$$

respectively. The capacitances were selected to be relatively close to their nominal values. The circuit was fed from the driving stage arrangement shown in Fig. 4.2, with switch closure time $\tau = 1 \mu \text{ sec.}$ The only adjustments made were the tuning of the four transmission zeros and the two resonant frequencies ω_{01} and ω_{02} according to the following scheme:

L_2, C_2	transmission zero	at	22.65 kHz
L_3, C_3	" "	"	15.605 kHz
L_4, C_6	" "	"	20.739 kHz
L_5, C_8	" "	"	14.763 kHz
L_1, C_1	parallel resonance	at	18.104 kHz
L_6, C_{10}	series resonance	at	17.424 kHz .

Measured attenuation curves are shown in Figs. 4.10 and 4.11. For almost all the stopband, there is good agreement between the calculated and measured attenuation, as shown in Fig. 4.10. The detailed attenuation curve measured in the passband is shown in Fig. 4.11. With respect to the reference frequencies of 16.8 kHz (CCITT) or 17 kHz (USA) (corresponding to voice frequencies of 800 Hz or 1000 Hz, respectively), the distortion does not exceed -0.1 dB anywhere, and +0.1 dB in most of the passband. Even at the cutoff frequencies (16.3 and 19.4 kHz), the deviation remains very moderate (0.6 dB), and this although no particular

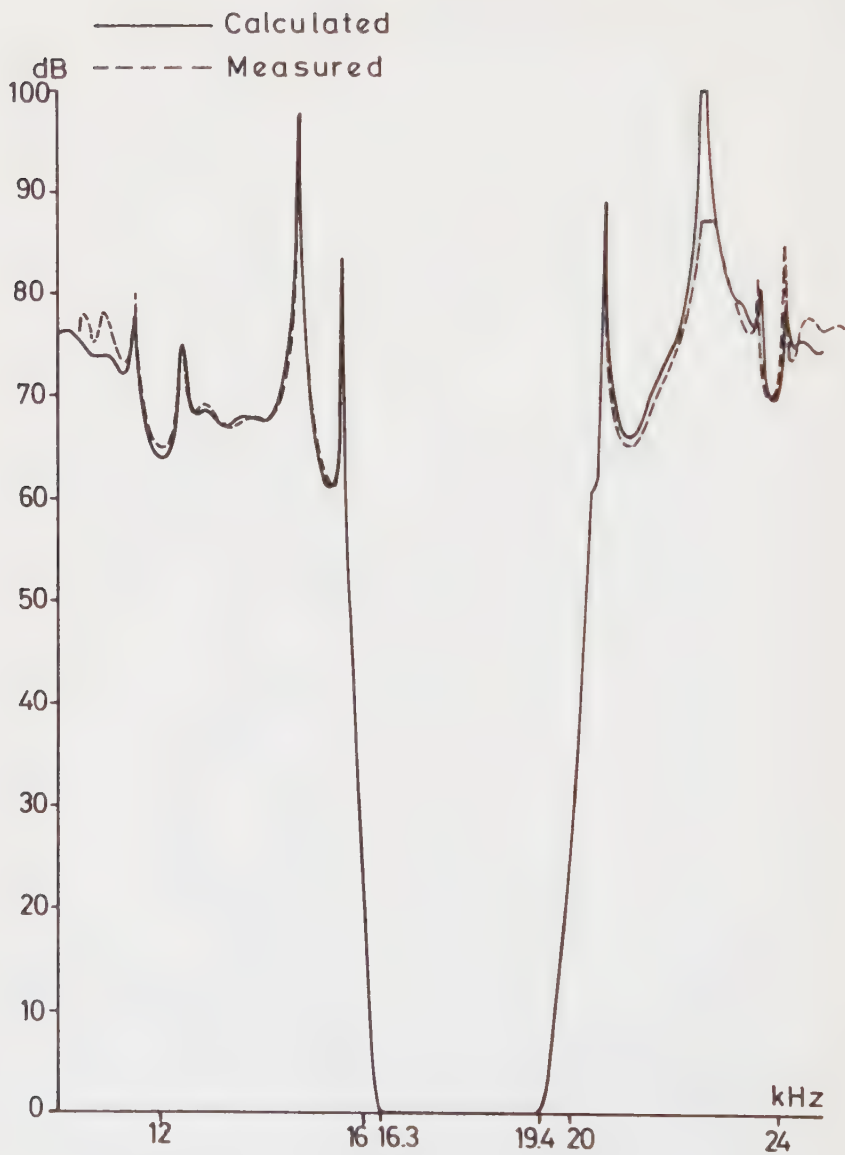


Fig. 4.10 Measured attenuation curve



Fig. 4.11 Measured attenuation curve in the pass-band

effort was made to further reduce the passband distortion.

4.3.2 Sensitivity analysis:

The optimum PCM/FDM conversion circuit was tested for its sensitivity behavior. Throughout the analysis, the locations of the transmission zeros, and the two resonant frequencies ω_{01} and ω_{02} of equation (4.7) were held constant. Whenever any of the capacitances C_1, C_2, C_4, C_6, C_8 or C_{10} were changed, the corresponding coil was also changed so that the resonant frequency and consequently the locations of the transmission zero and of ω_{01} and ω_{02} were not altered. To demonstrate the effect of varying an element, the ideal attenuation curve is compared with the new resulting one. In Fig. 4.12, the effect of varying the terminating resistance R_2 (the least critical element) by $\pm 5\%$ is shown. Another curve, in Fig. 4.13, shows the effect of varying the capacitance C_6 (the most critical element) by $\pm 5\%$.

As is shown in the curves of Figs. 4.12 and 4.13, the minimum sensitivity property is manifested. Again, the demonstrated results are so chosen to make the effect of element variations visible on the given curves. For variations of the order of $\pm 1\%$ in element values, the corresponding deviations of the attenuation were found to be noticeably smaller than one fifth of those obtained for $\pm 5\%$ element changes.

4.3.3 Comparison with doubly-terminated networks:

To compare the obtained results with those obtained for a doubly-terminated filter, a 12th order doubly-terminated filter was computed having the following properties [10]:

f_{C1}	= 16.3 kHz	lower cut-off frequency
f_{C2}	= 19.4 kHz	upper cut-off frequency
A_d	= 0.1 dB	maximum allowable attenuation in passband
f_{S1}	= 15.7 kHz	lower stopband edge
f_{S2}	= 20.6 kHz	upper stopband edge
A_{S1}	= 54.4 dB	lowest attenuation in lower stopband
A_{S2}	= 62.1 dB	lowest " " higher "

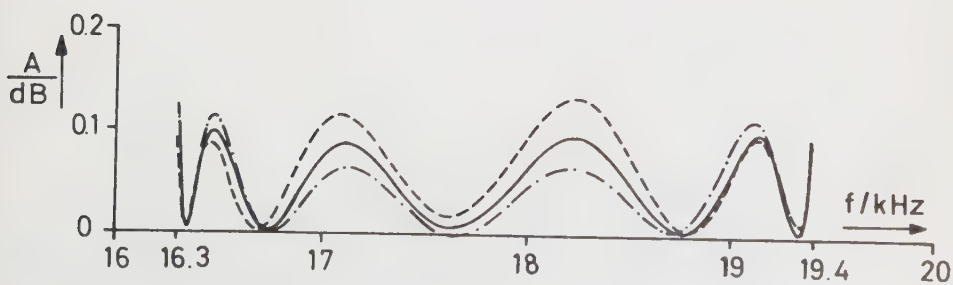
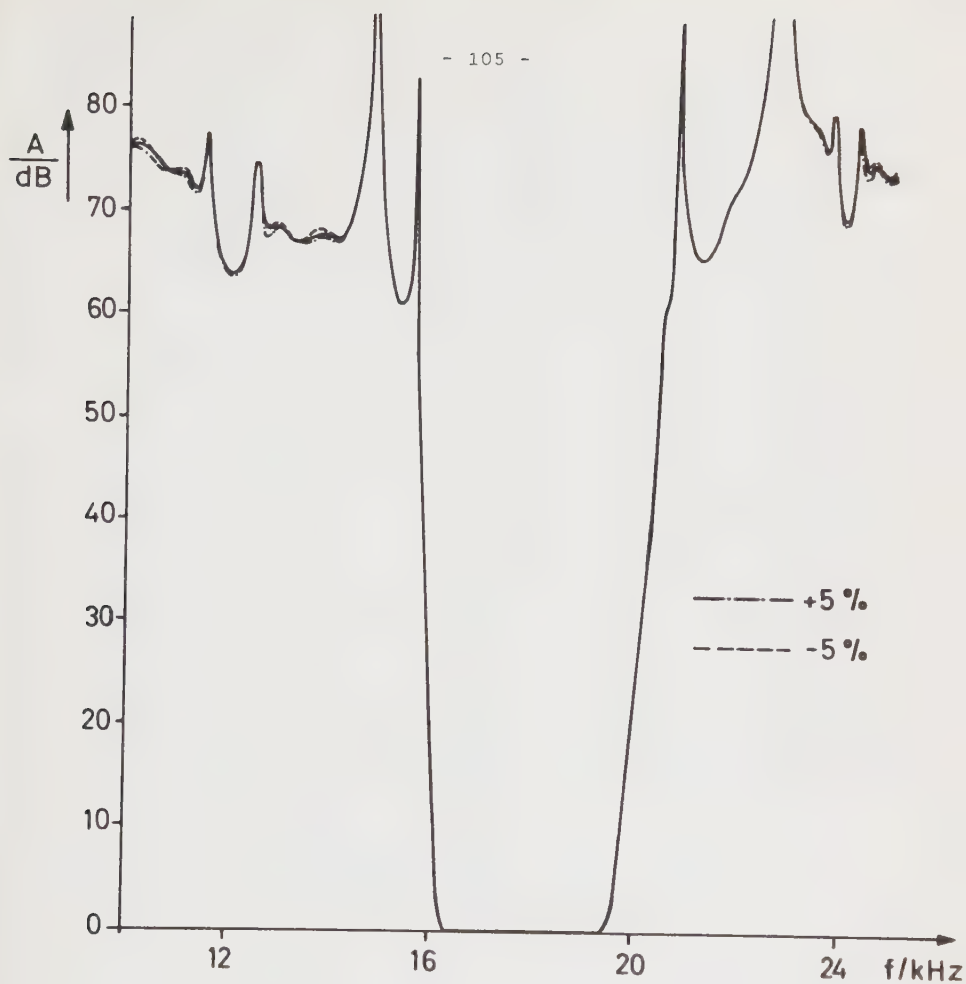


Fig. 4.12 Effect of varying R_2 by $\pm 5\%$

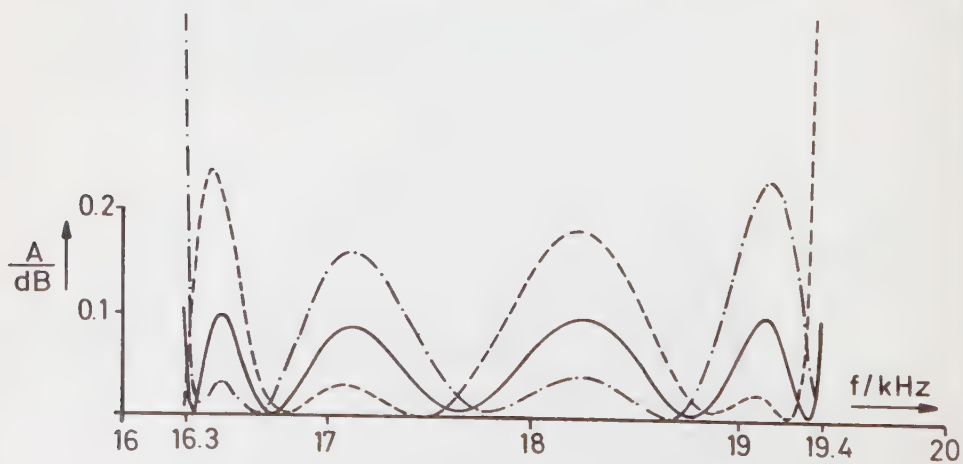
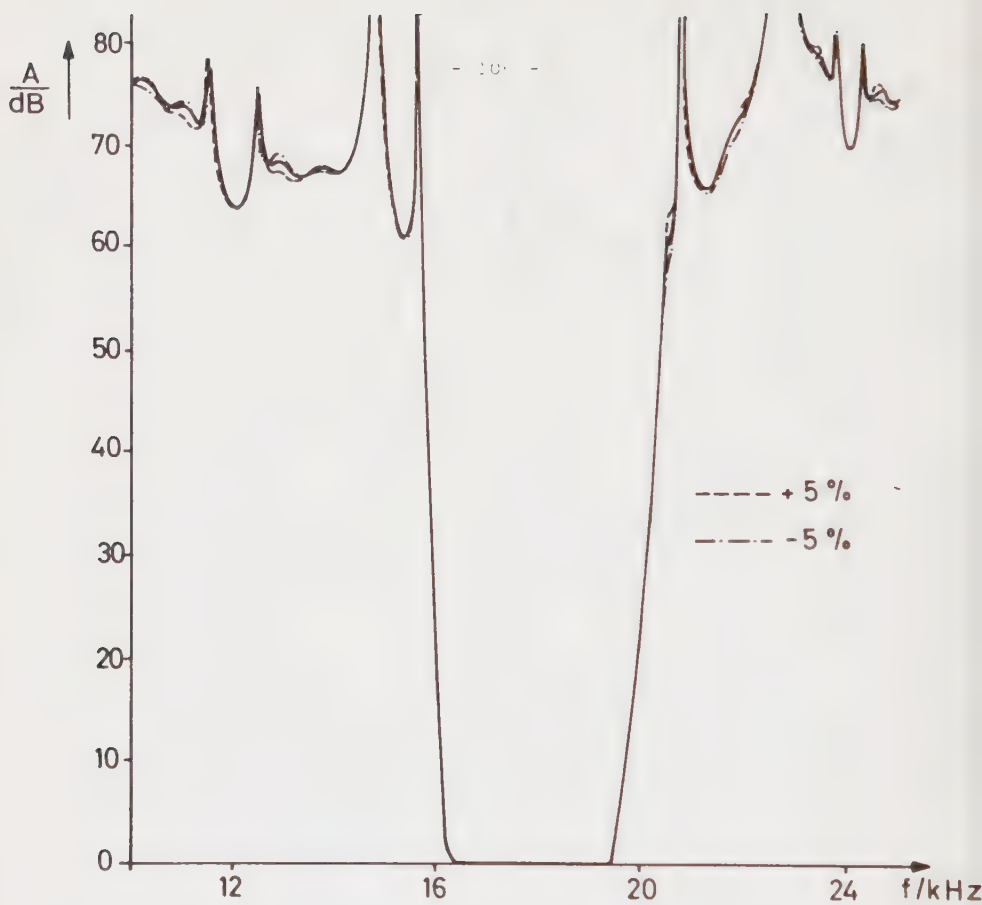


Fig. 4.13 Effect of varying C_1 by $\pm 5\%$

It is seen that the highest attainable values thus obtained for A_{S1} and A_{S2} are still lower than those obtained for the conversion circuit (60 and 65 dB). The comparison is also shown in the curves of Fig. 4.14 where both attenuation curves are drawn. The improvement is thus apparently somewhat smaller than in the lowpass case. This however may be due to the fact that in the solution obtained for the conversion filter the two attenuation poles which are most remote from the passband are much further away than for the doubly-terminated filter. The minimum stopband attenuation of the conversion filter could clearly be raised by shifting its most remote attenuation poles close to the stopband edges (with the hope that the attenuation in the passband remains lower than A_d). A final judgement is usually difficult in such situations where the optimum solution is obtained numerically. It is extremely difficult to prove that any obtained minimum in a numerical minimization process is actually the global minimum rather than being merely a local one.

We conclude by stating that it is not possible to make a comparison of the results obtained in the bandpass case with other results. No circuits and/or corresponding results for PCM/FDM direct conversion have yet appeared in any publication.

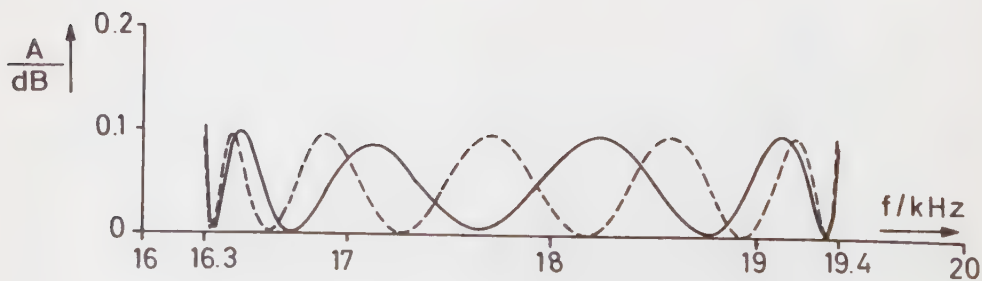
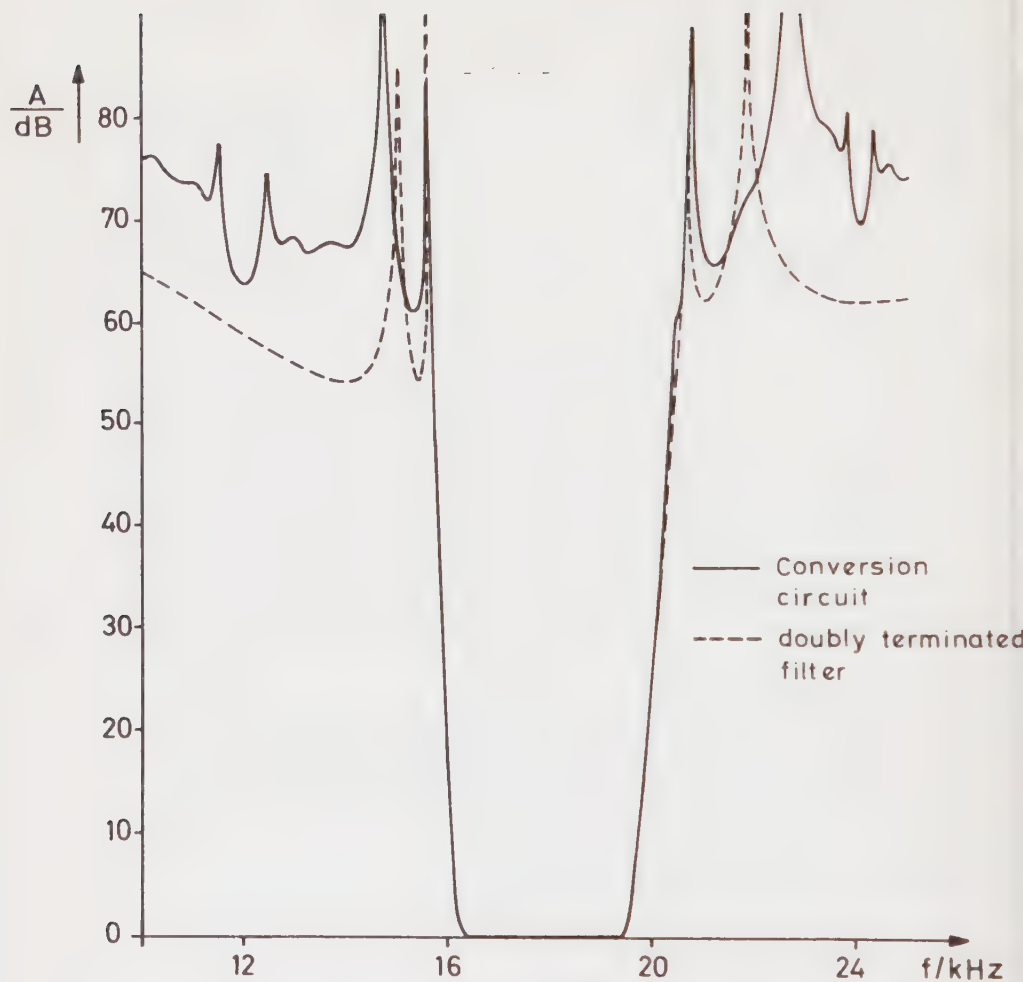


Fig. 4.14 Comparison between conversion filter and a doubly-terminated filter of the same order

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Appendix 1

In the light of the analysis and the results obtained for the conversion circuits, especially the conditions under which it is possible to design conversion circuits with a maximum allowable attenuation A_d in the passband, we suggest another algorithm to design conversion circuits. The algorithm uses the formal similarity of equation (2.96) and (3.14), for the conversion coefficient $\Sigma_o(p)$ and the transfer function $T(p)$ of a doubly-terminated filter, which are rewritten here for convenience

$$\Sigma_o(p) = 2 \sqrt{R_C R_2} \frac{M(p)}{R_C + \tilde{Z}(p)} \quad (A.1)$$

$$T(p) = 2 \sqrt{R_1 R_2} \frac{M(p)}{R_1 + Z_1(p)} \quad (A.2)$$

The reasons why it is difficult to use doubly-terminated filter circuits to build conversion circuits are the differences between R_1 and R_C on one hand and $\tilde{Z}(p)$ and $Z_1(p)$ on the other hand. Both R_C and $\tilde{Z}(p)$ depend on the circuit element values (circuit parameters) and the sampling frequency.

To overcome this difficulty one may achieve an apparent equality between R_C and R_1 by defining the auxiliary frequency F_r , where

$$F_r = 1/T_r, \quad (A.3)$$

and for a doubly-terminated filter circuit whose input impedance Z_1 shows an impedance which reduces to a capacitance C_1 at high frequency according to equation (2.29), T_r is chosen such that

$$R_C = T_r / 2C_1 = R_1. \quad (A.4)$$

Similarly, an apparent approximate equality is achieved between $Z_1(p)$ and $\tilde{Z}(p)$ by choosing a second auxiliary frequency F_z , where

$$F_z = 2\pi / \Omega_z. \quad (A.5)$$

and

$$\tilde{Z}(p) = \sum_{n=-\infty}^{\infty} Z_1(p + jn\Omega_z) = Z_1(p). \quad (A.6)$$

The choice of Ω_Z for achieving the approximate equality (A.6) is made by satisfying

$$F_Z \gg f_C \quad (A.7)$$

where f_C is the cutoff frequency of the filter (see reference [6] of chapter 3).

Now, the apparent equality between (A.1) and (A.2) is achieved and the behavior of an "auxiliary" conversion circuit is similar to that of a doubly-terminated filter. It is expected that both F_r and F_Z are different from the actual sampling frequency F . Through the aid of a doubly-terminated circuit (element values or circuit parameters) the design process can be started. The suggested algorithm must now update the circuit parameters and the values of the two auxiliary frequencies such that both F_r and F_Z reach the value F of the actual sampling frequency. During the updating process, the value of the maximum attenuation inside of the passband is kept equal to a specified value A_d while trying to keep the maximum attenuation in the stopband as high as possible.

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